Relatividade e Gravitação

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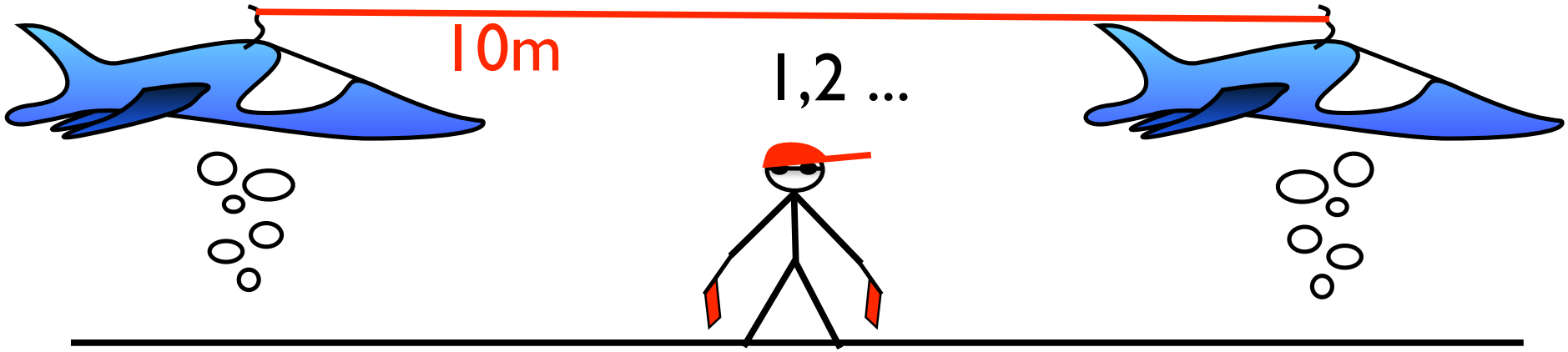
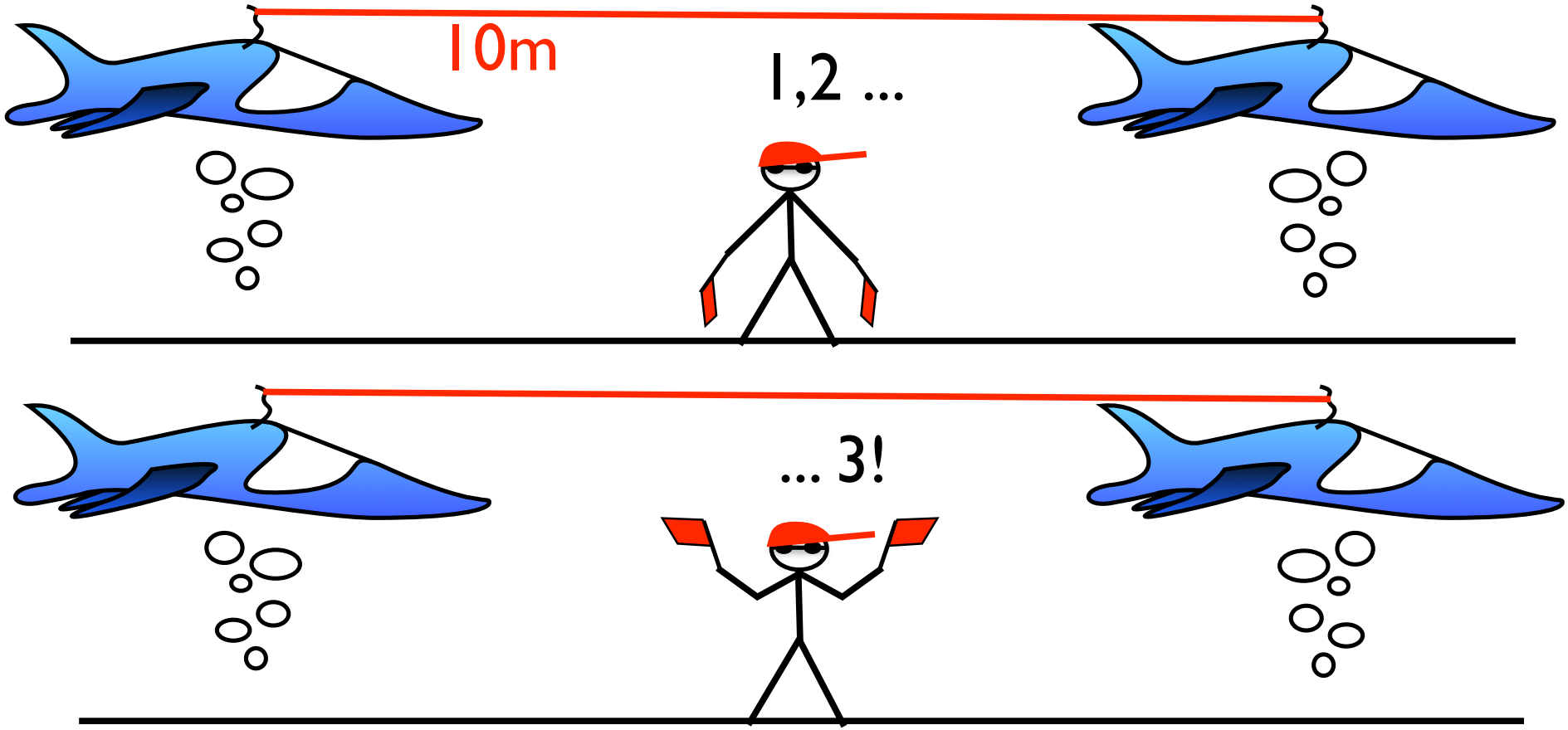
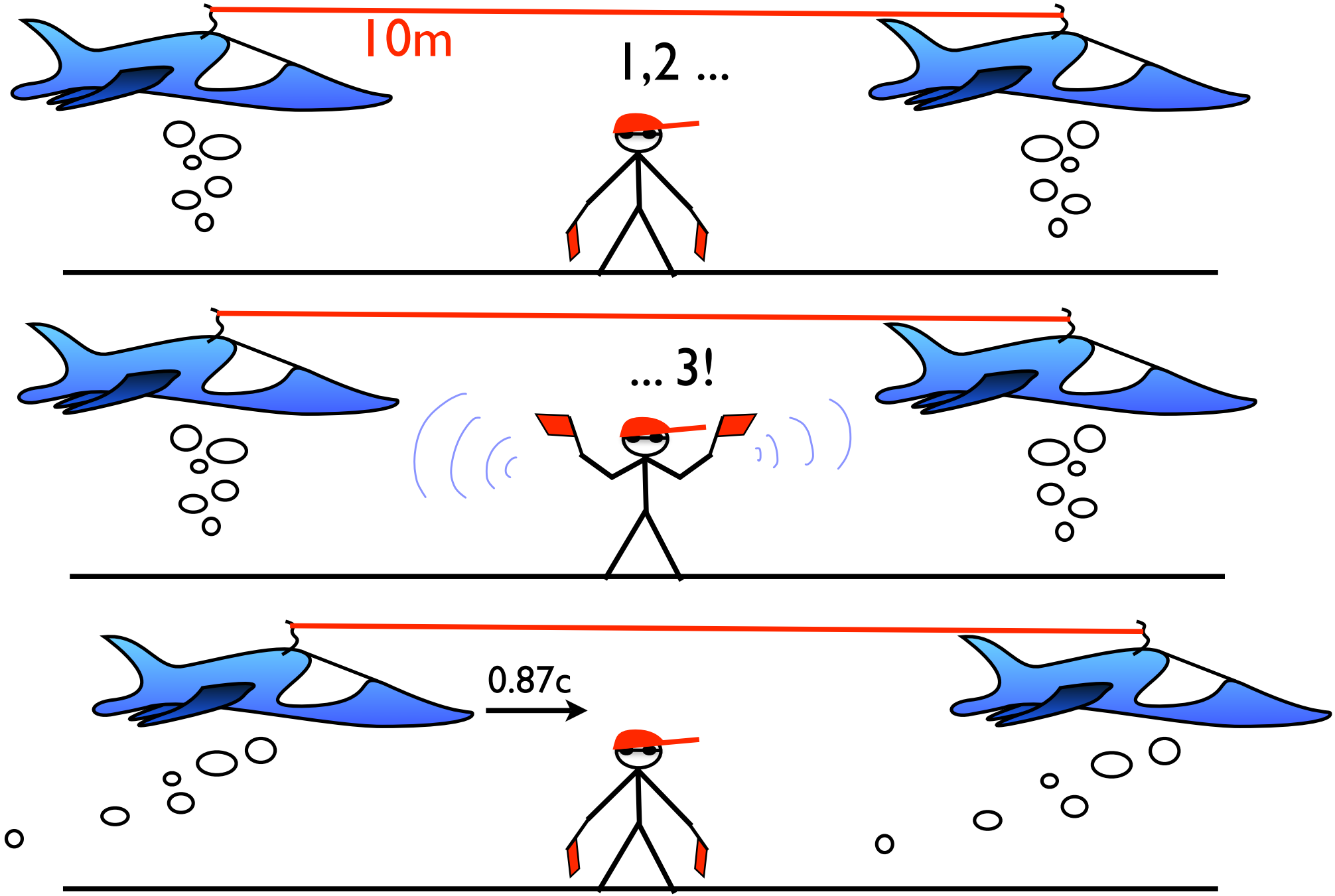
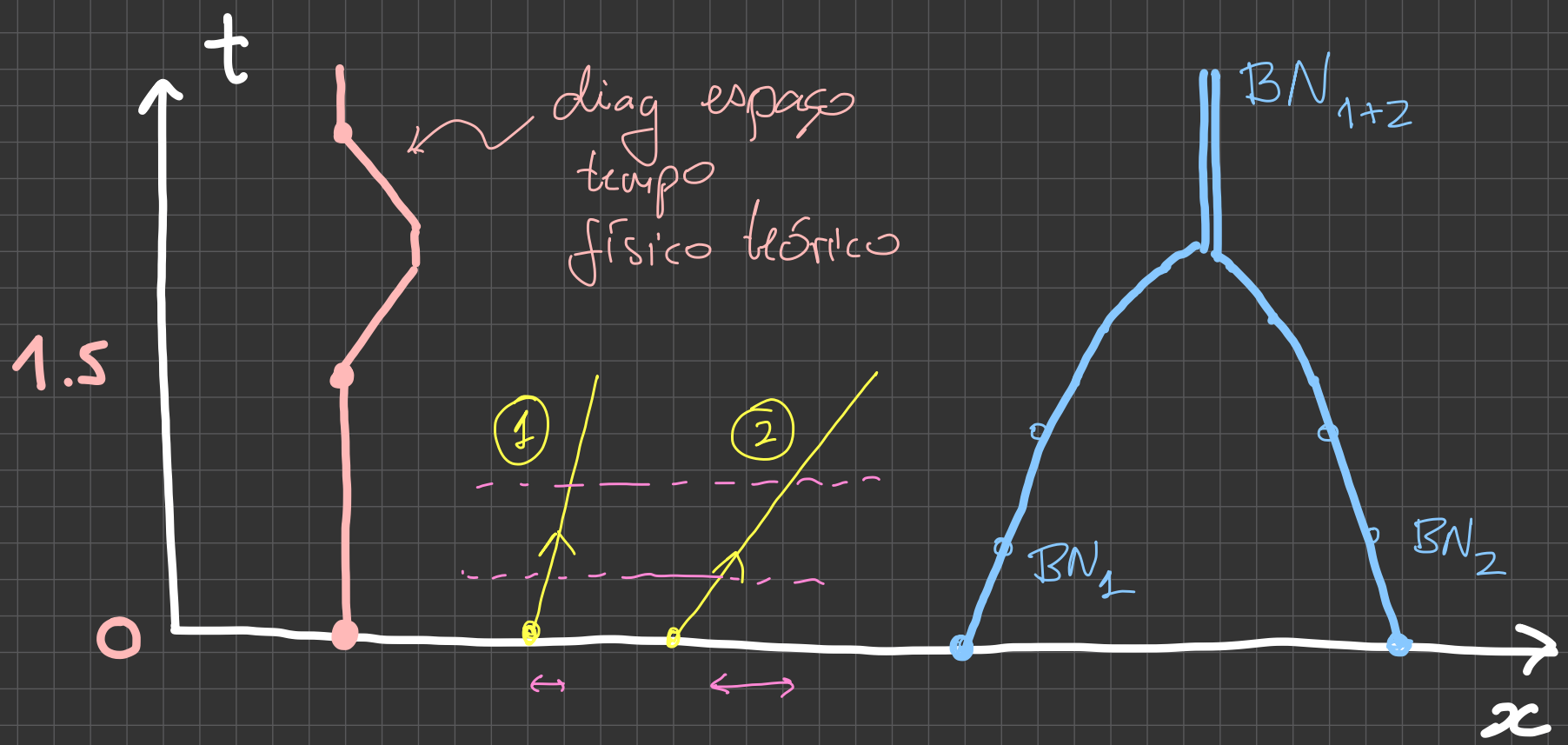
ICTP SAIFR | International Centre for Theoretical Physics
South American Institute for Fundamental Research


10m
1,2 ...

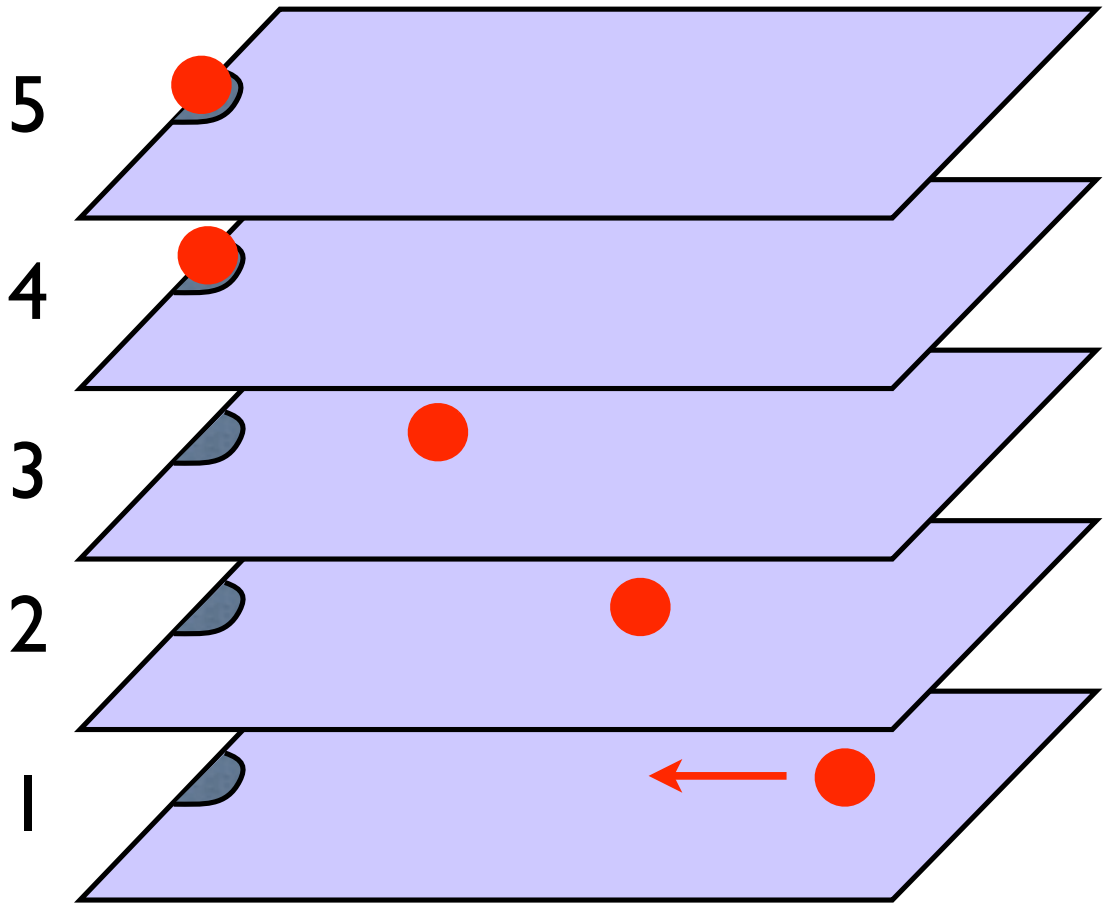

10m
1,2 ...
... 3!

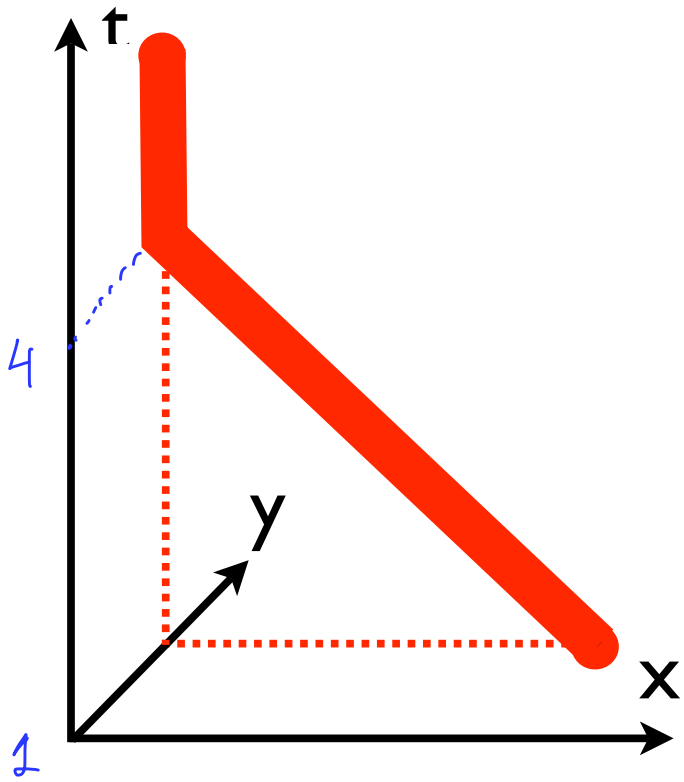

10m
1,2 ...
... 3!
0.87c

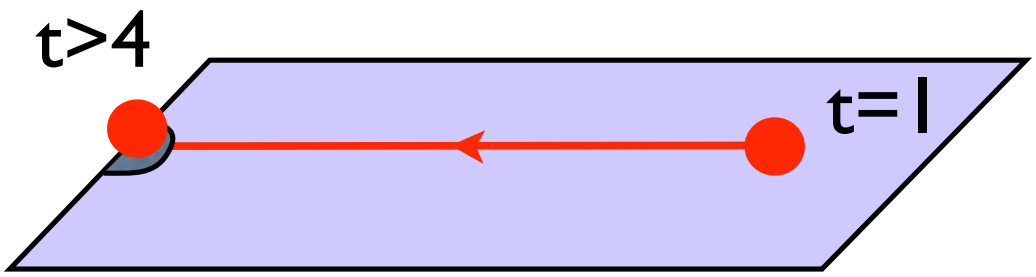
Diagramas de Espaço-Tempo

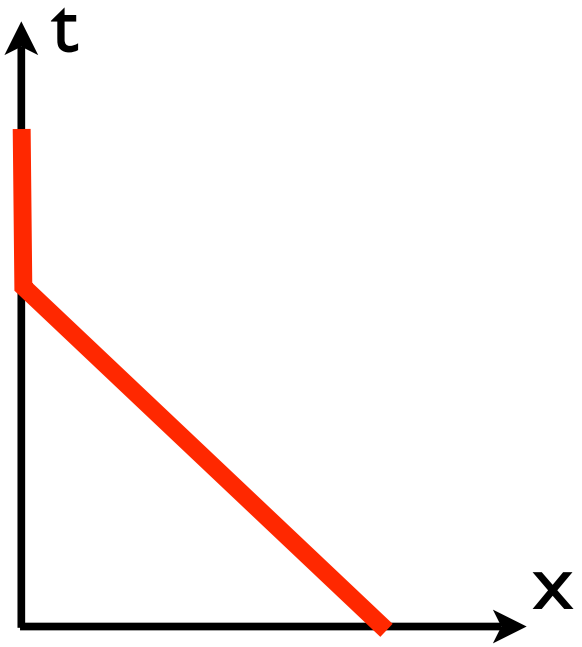

t
diag espaço tempo
físico teórico
1.5
0
1
2
BN1+2
BN1
BN2
x

VÁRIAS FOTOS :

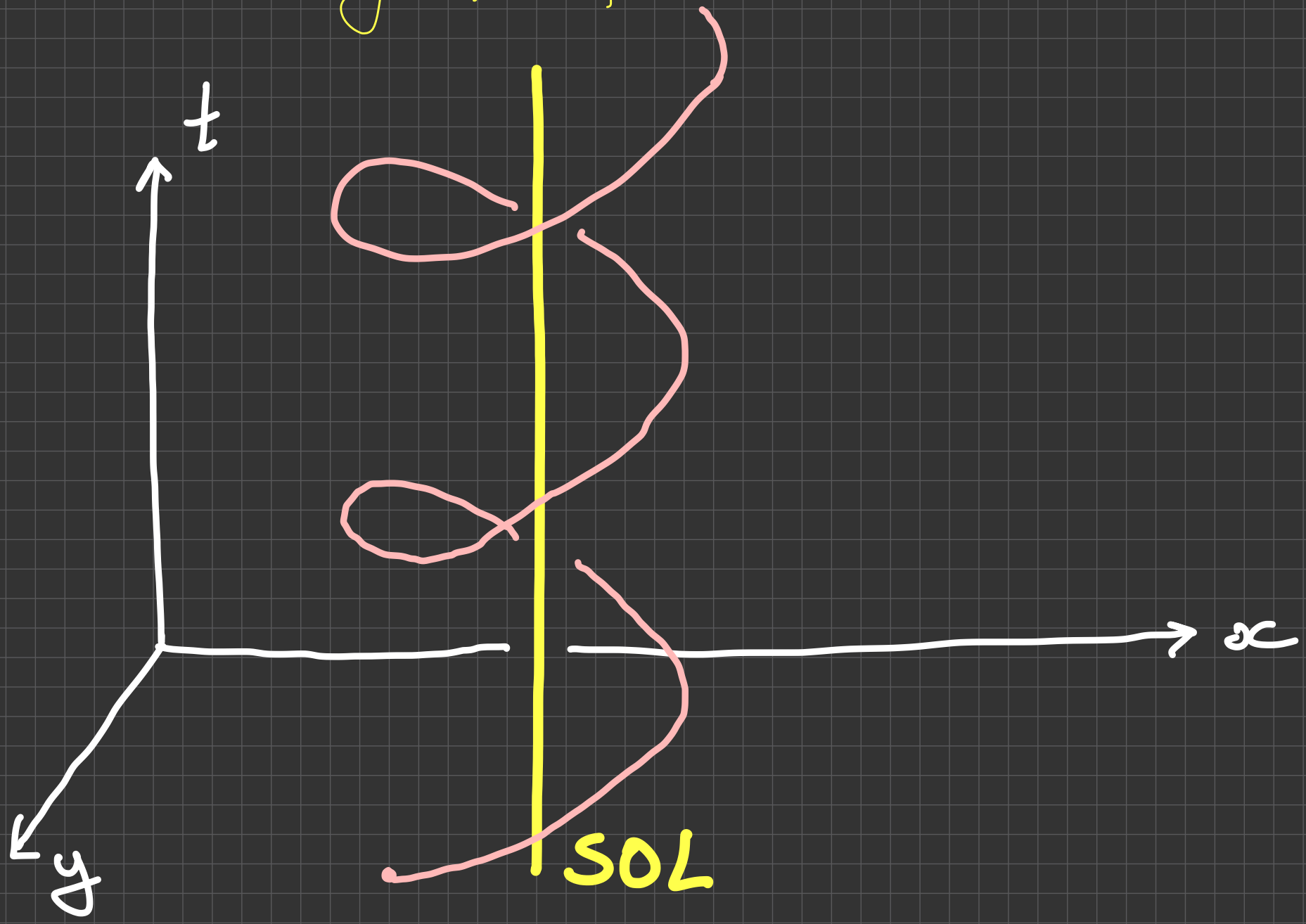


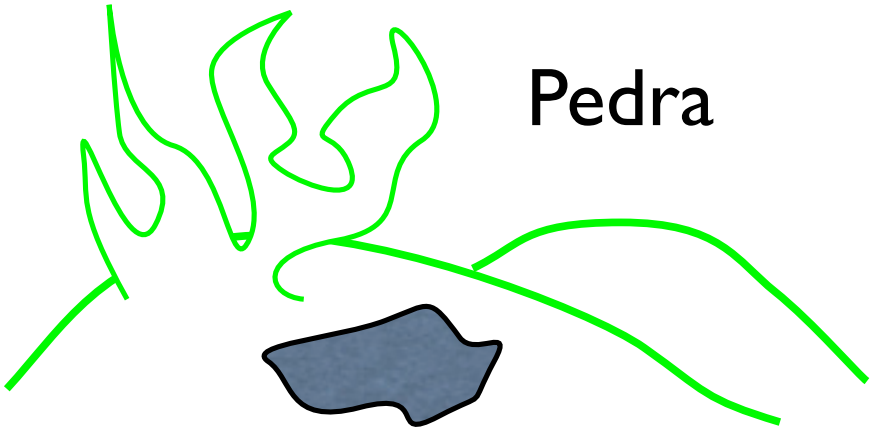


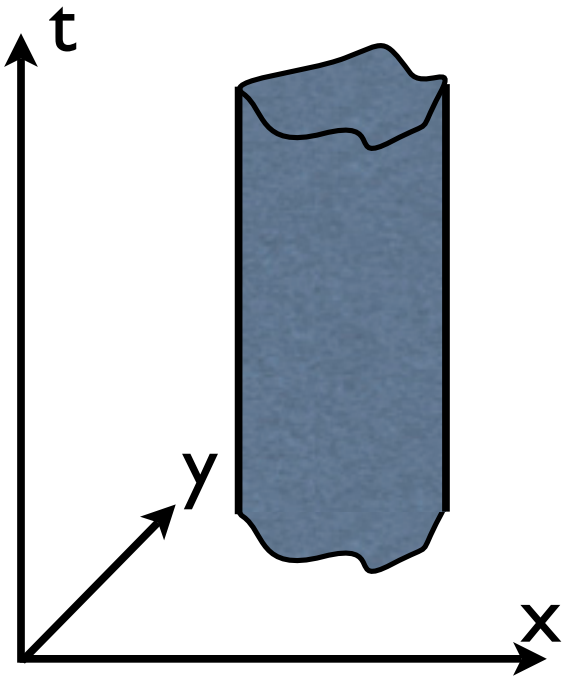




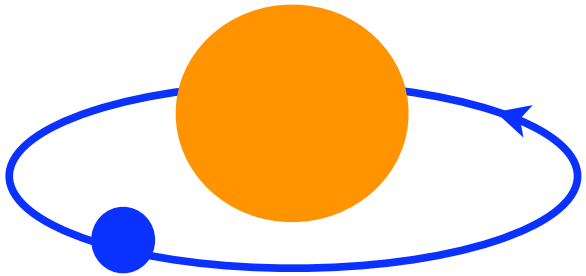
Q: diag p/ planeta em torno do sol

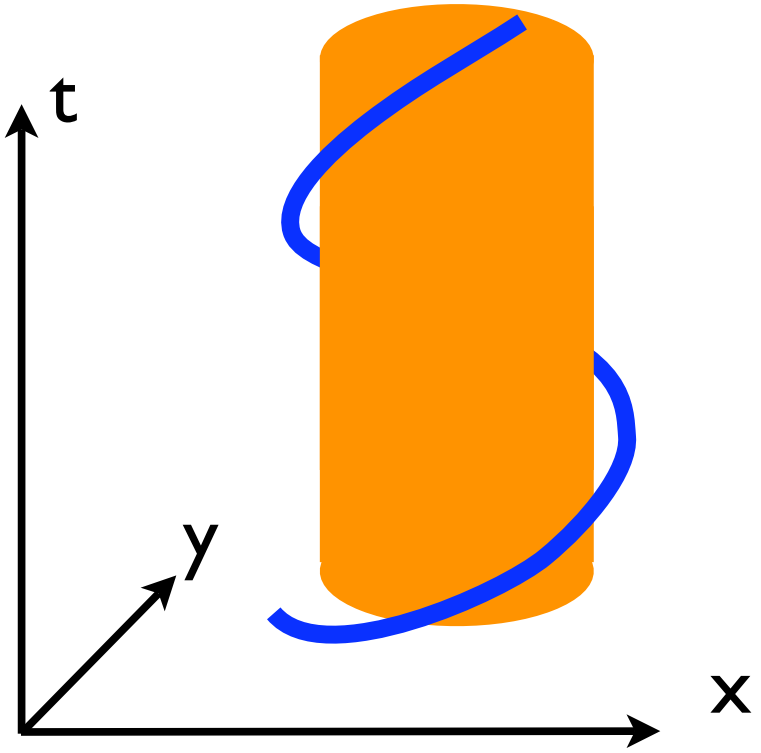




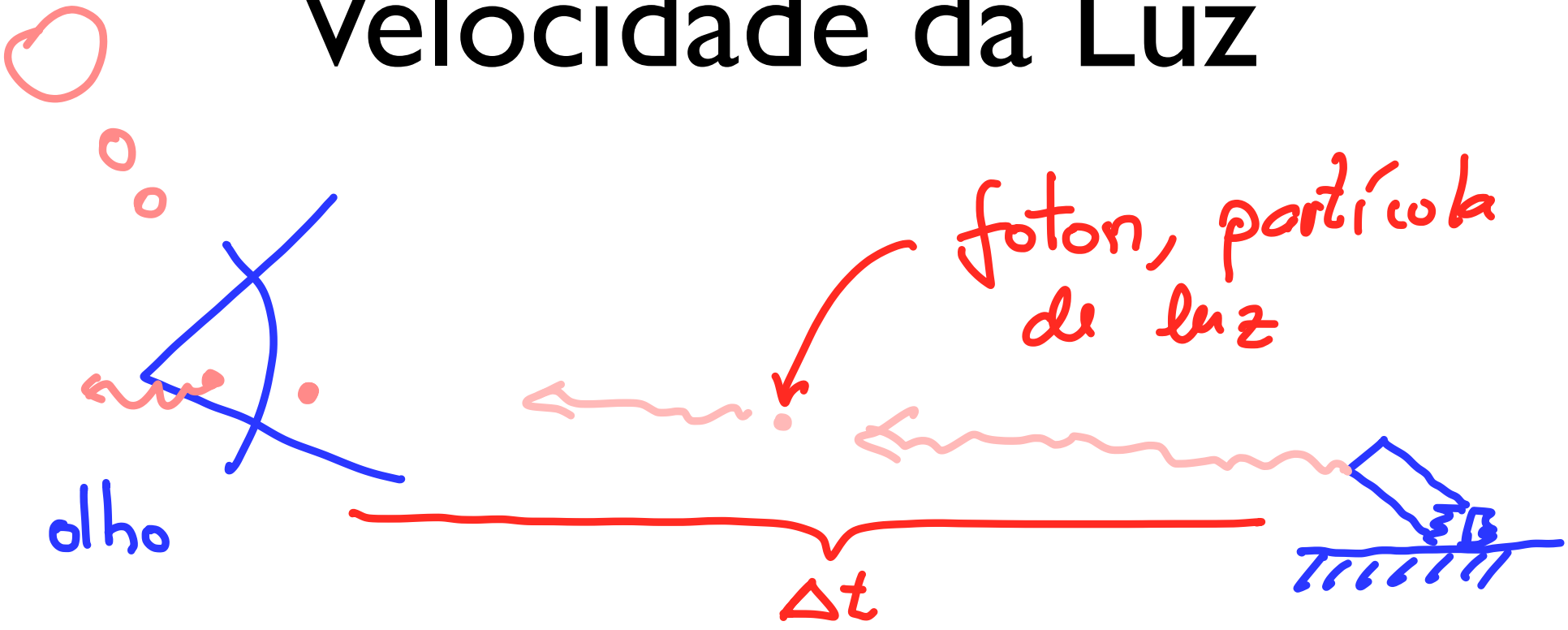


Planeta à volta de um Sol

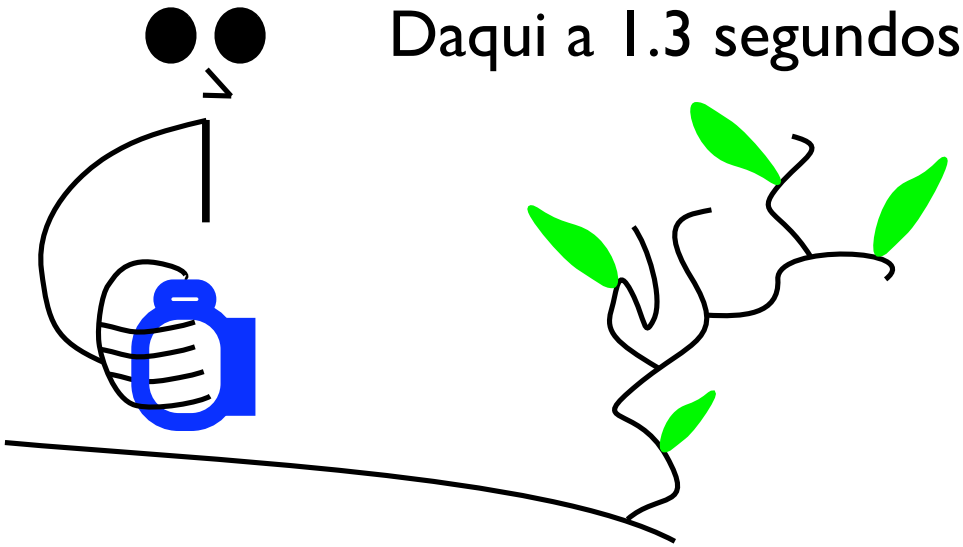


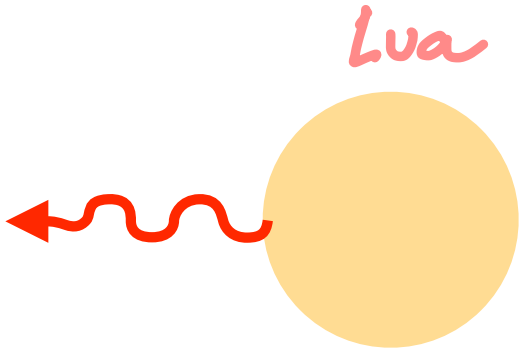


Velocidade da Luz

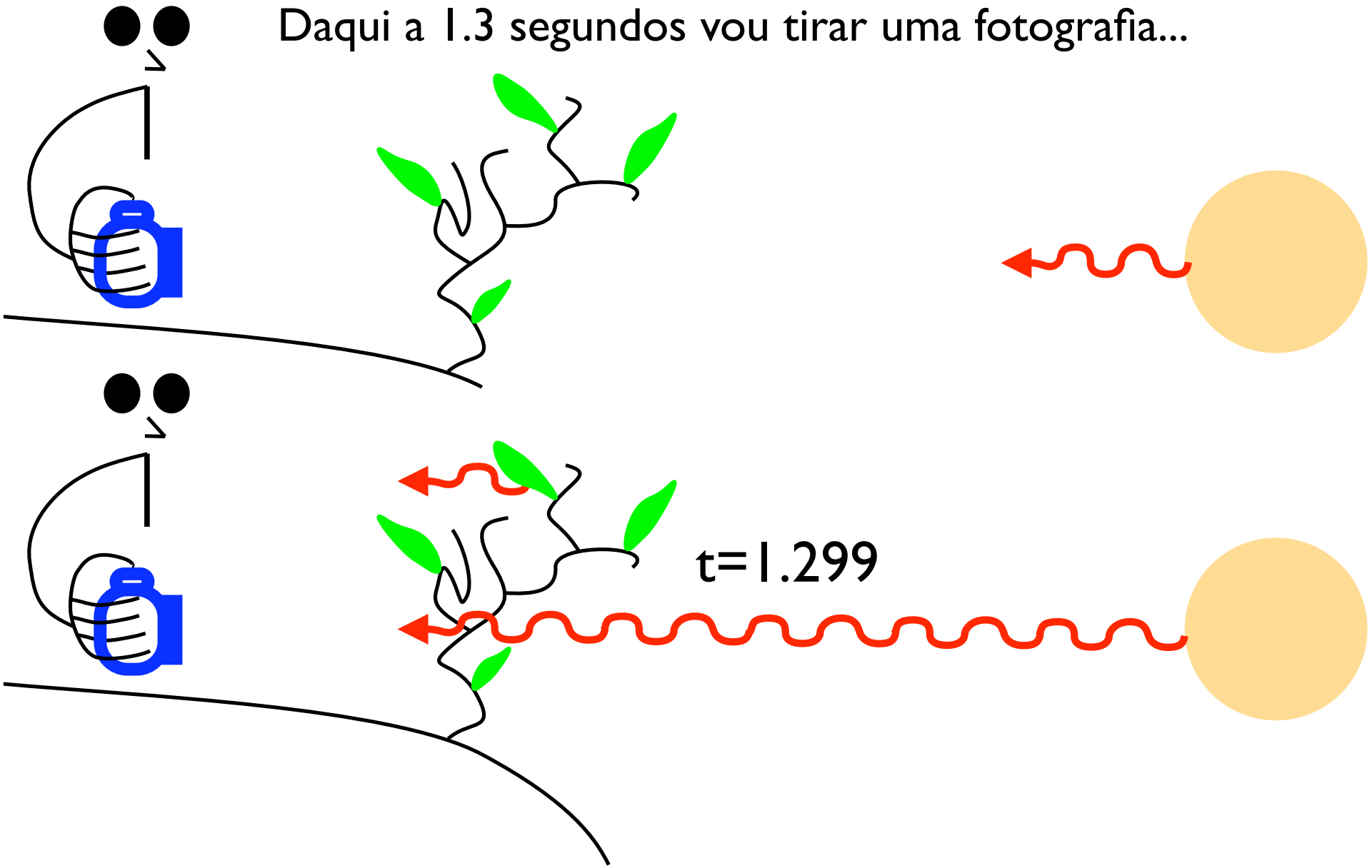


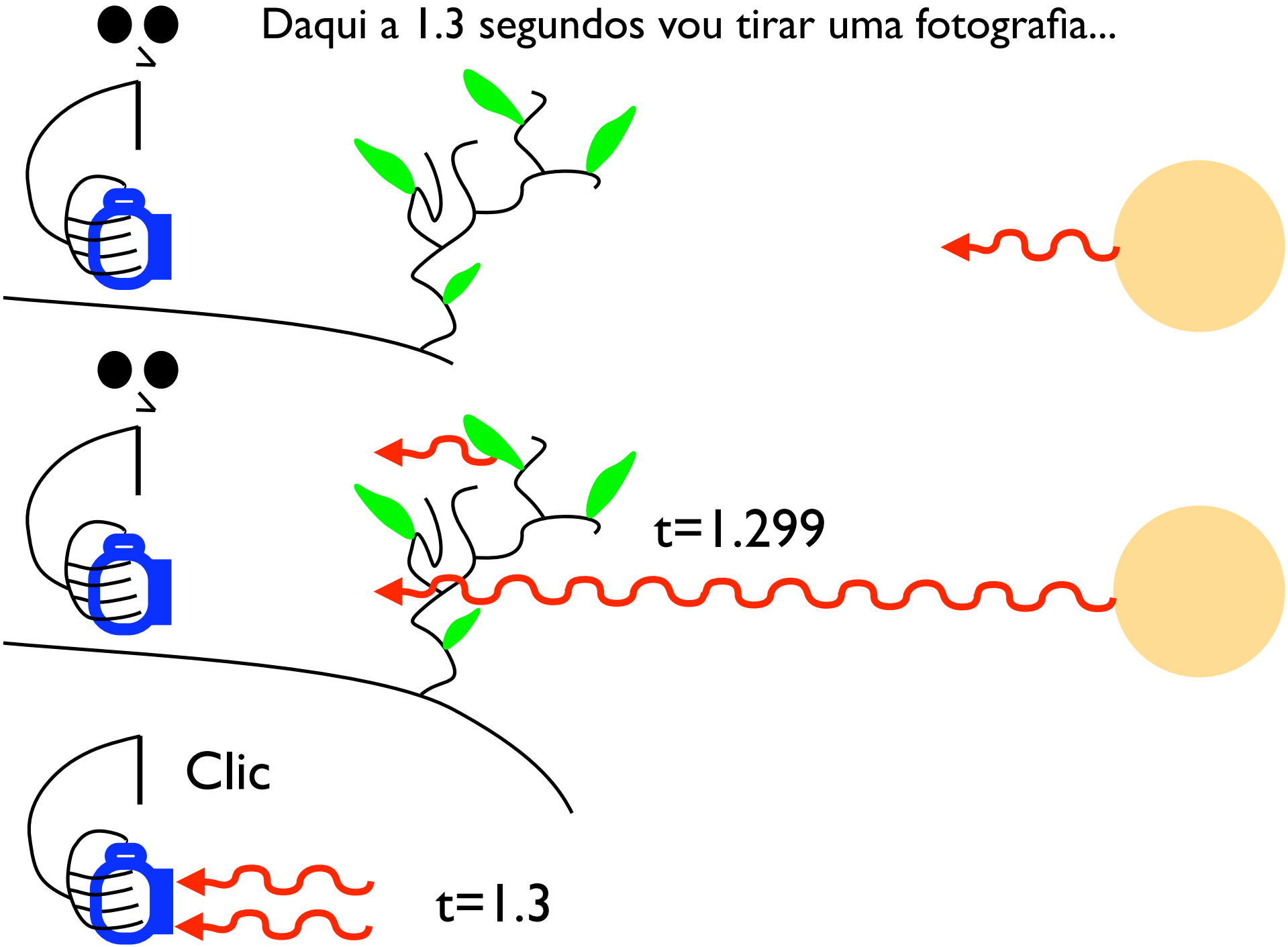
Daqui a 1.3 segundos vou tirar uma fotografia...





Daqui a 1.3 segundos vou tirar uma fotografia...




Daqui a 1.3 segundos vou tirar uma fotografia...
t=1.299
Clic
t=1.3

Fotografia

Lua há 1.3 segundos atrás

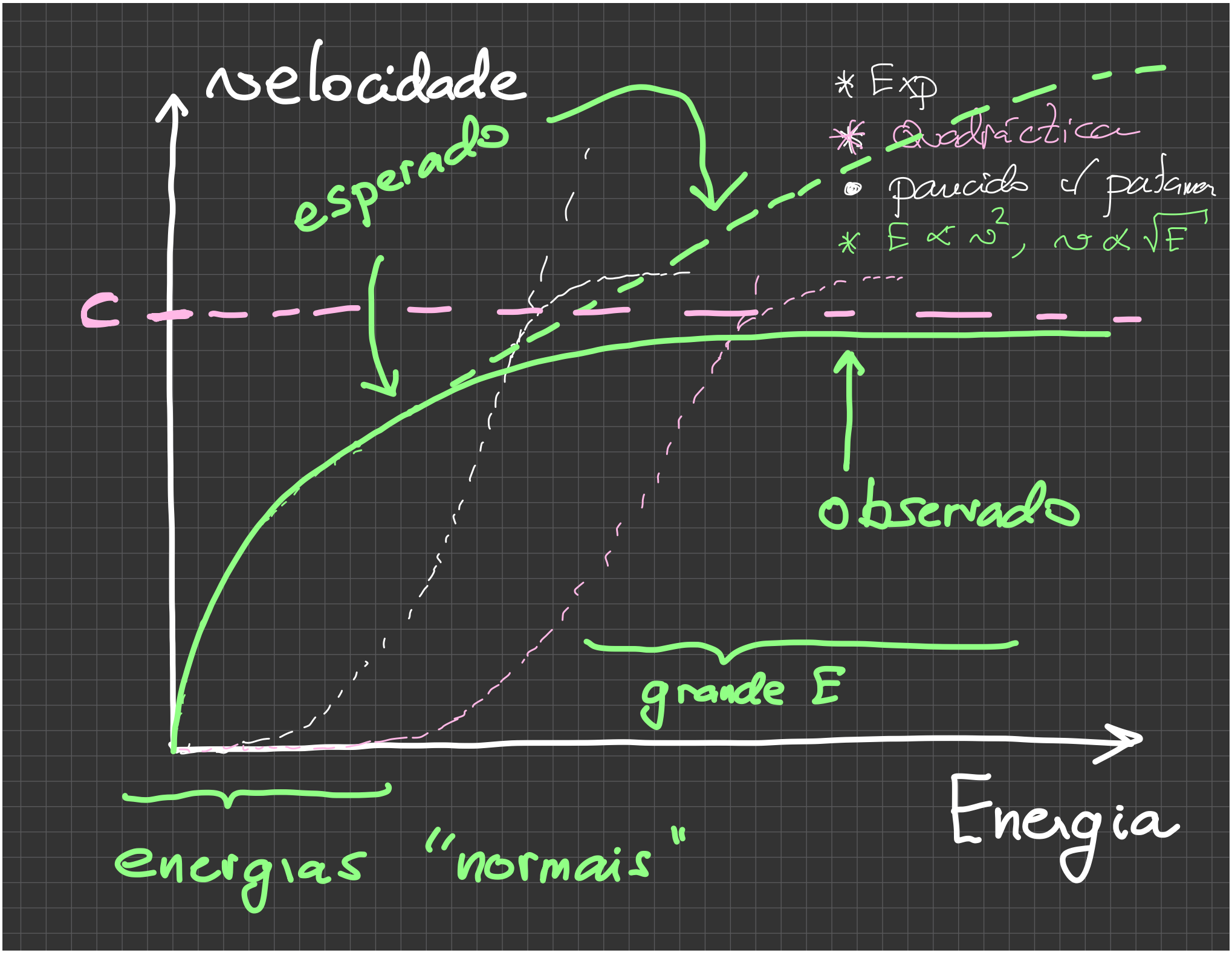
Planta há milionésimos de segundos

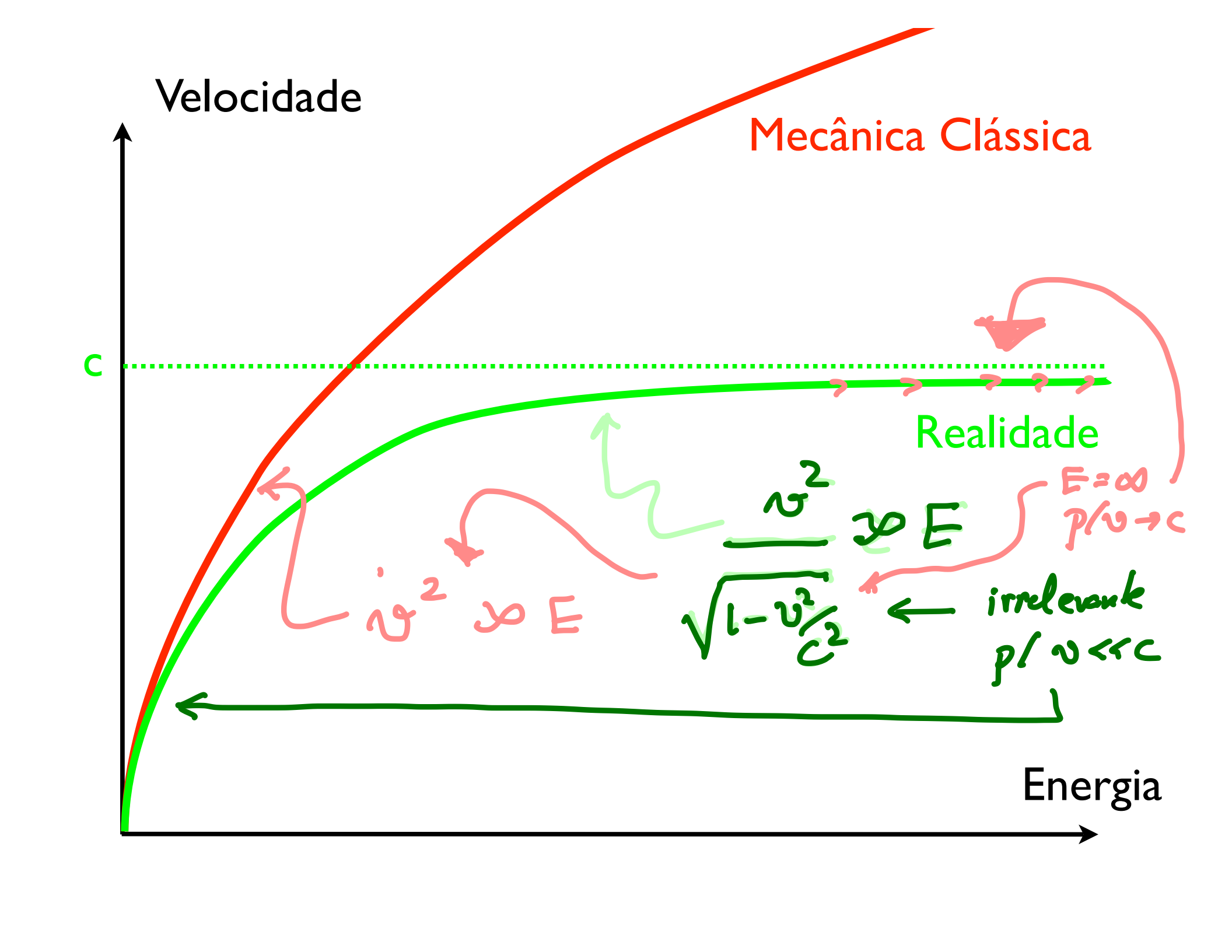
$$c = 300\,000\,000 \text{ m/s}$$

FACTO EXPERIMENTAL

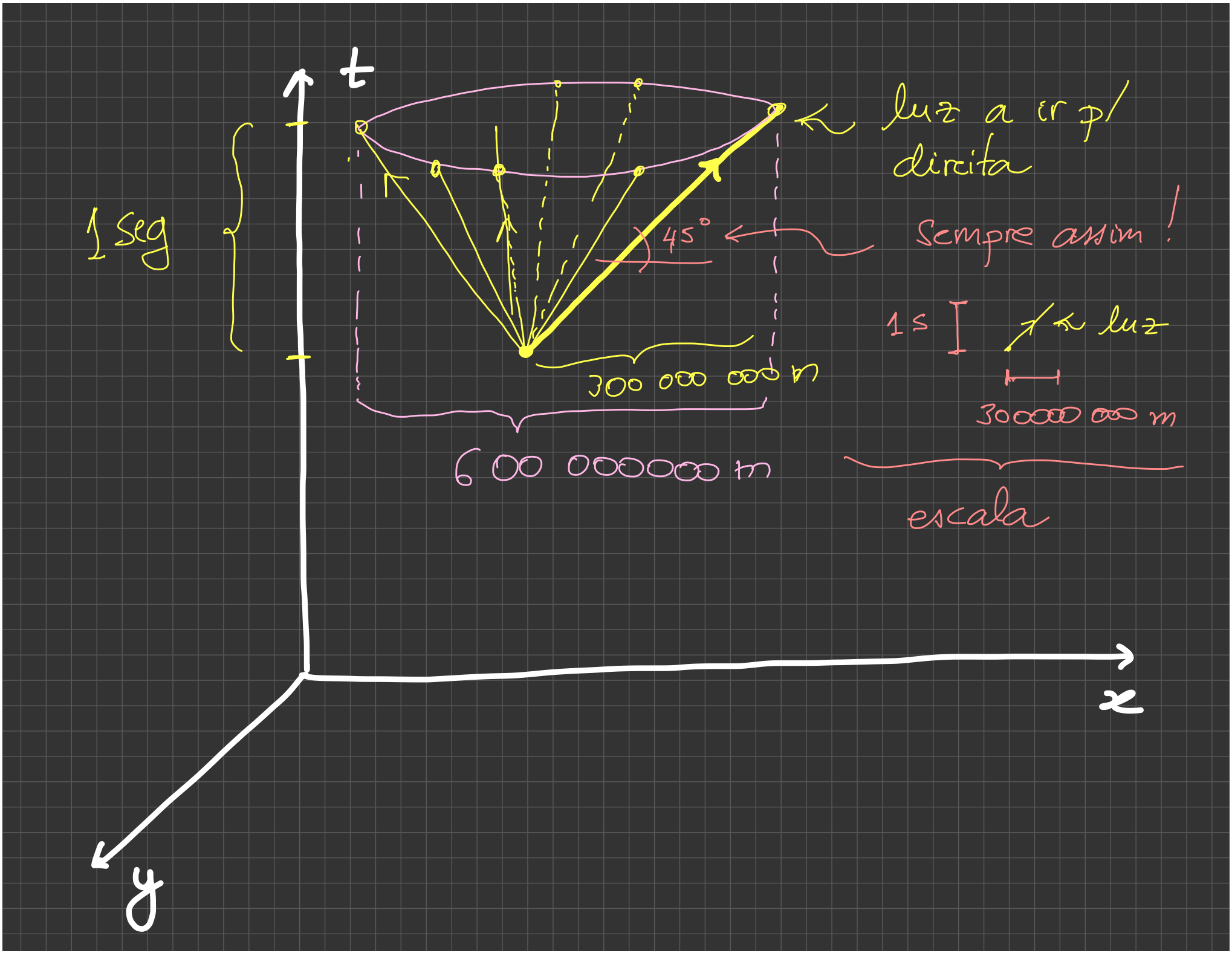
não dá pf passar c.

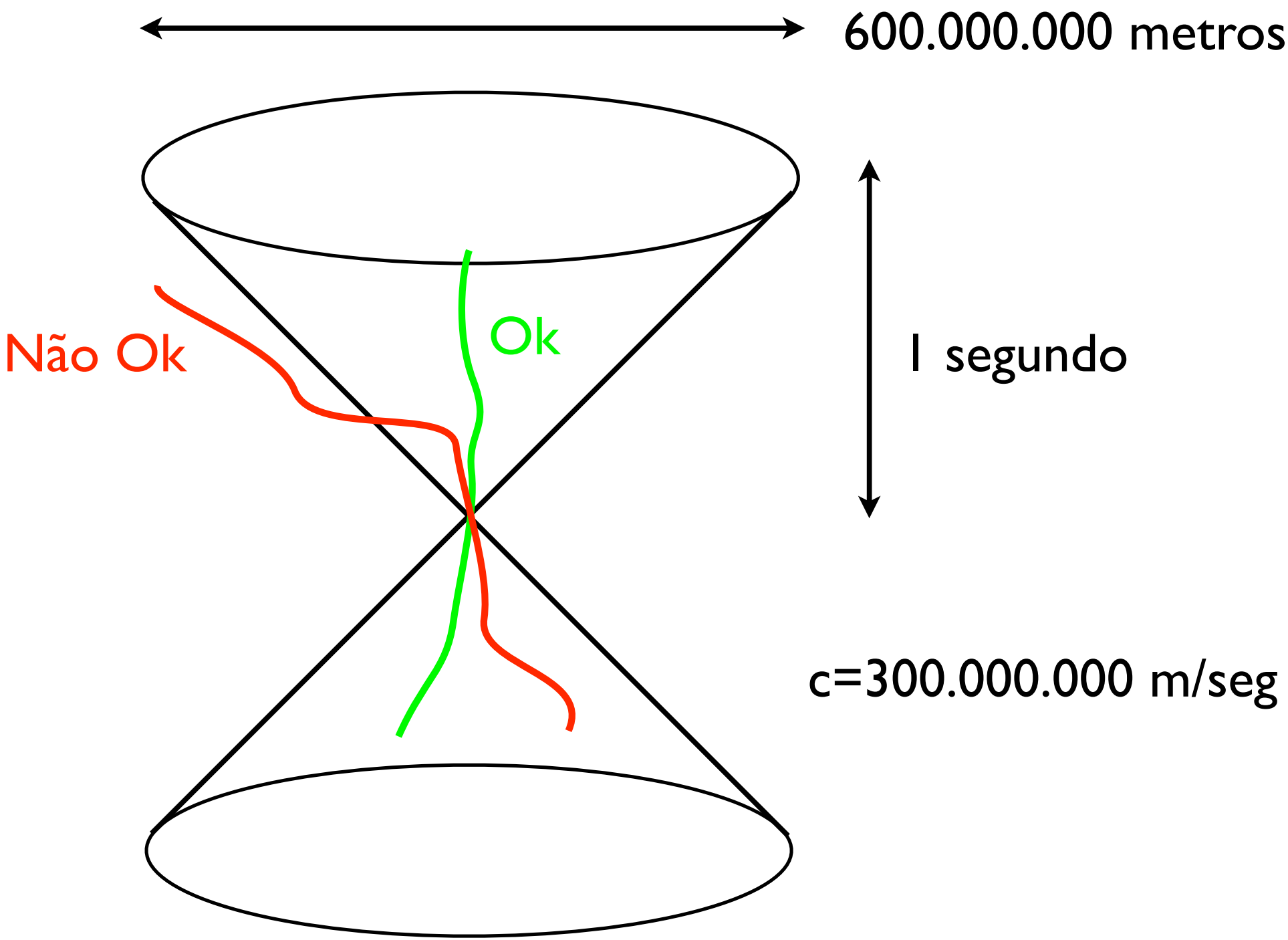
↑ velocidade da luz

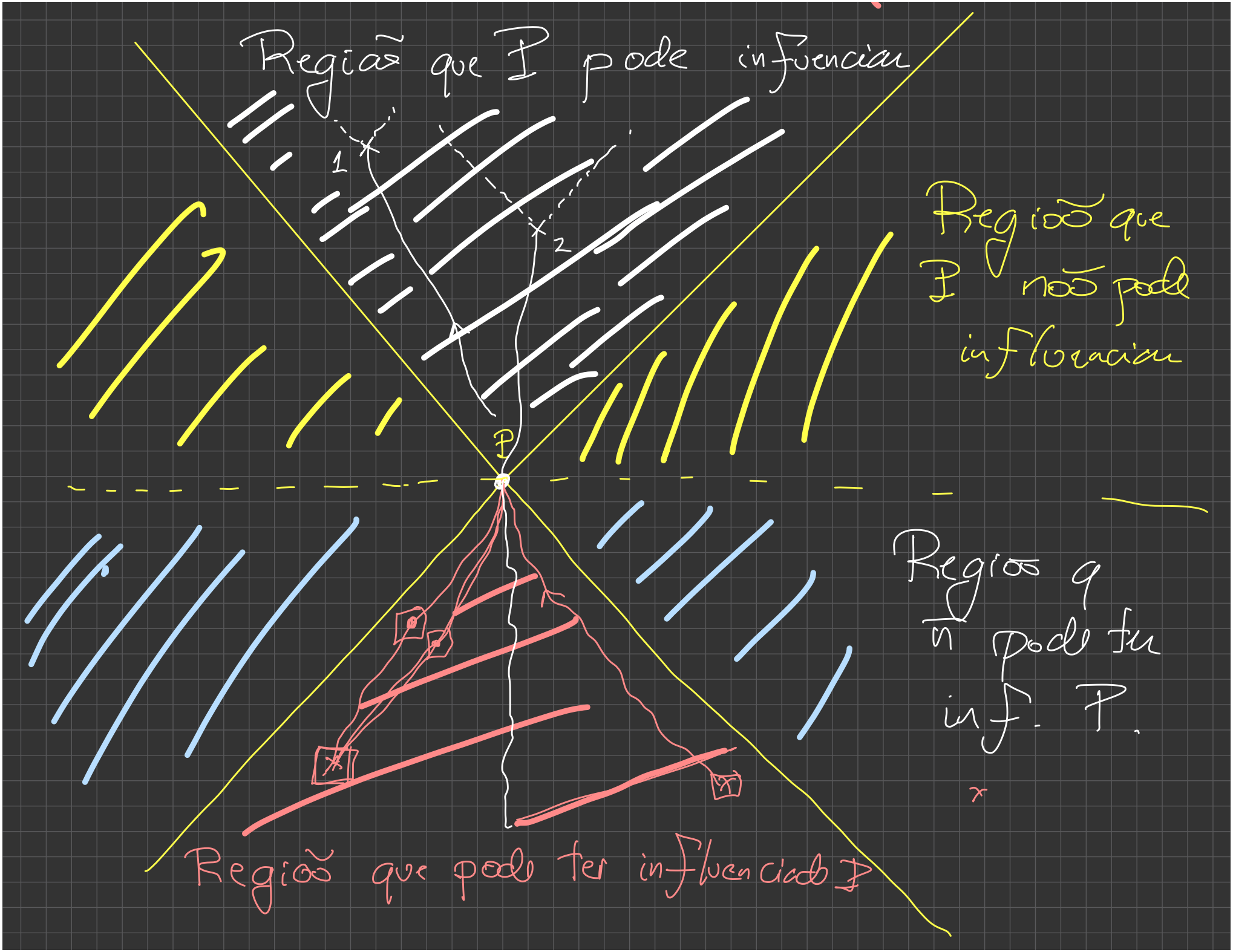

velocidade
esperado
* Exp
* quadrática
• parecido c/ partículas
* $E \propto v^2$, $v \propto \sqrt{E}$
c
observado
grande E
energias "normais"
Energia


Velocidade
Mecânica Clássica
c
Realidade
$v^2 \propto E$
$\frac{v^2}{\sqrt{1-v^2/c^2}} \propto E$
E=∞
p/ v→c
irrelevante
p/ v<<c
Energia

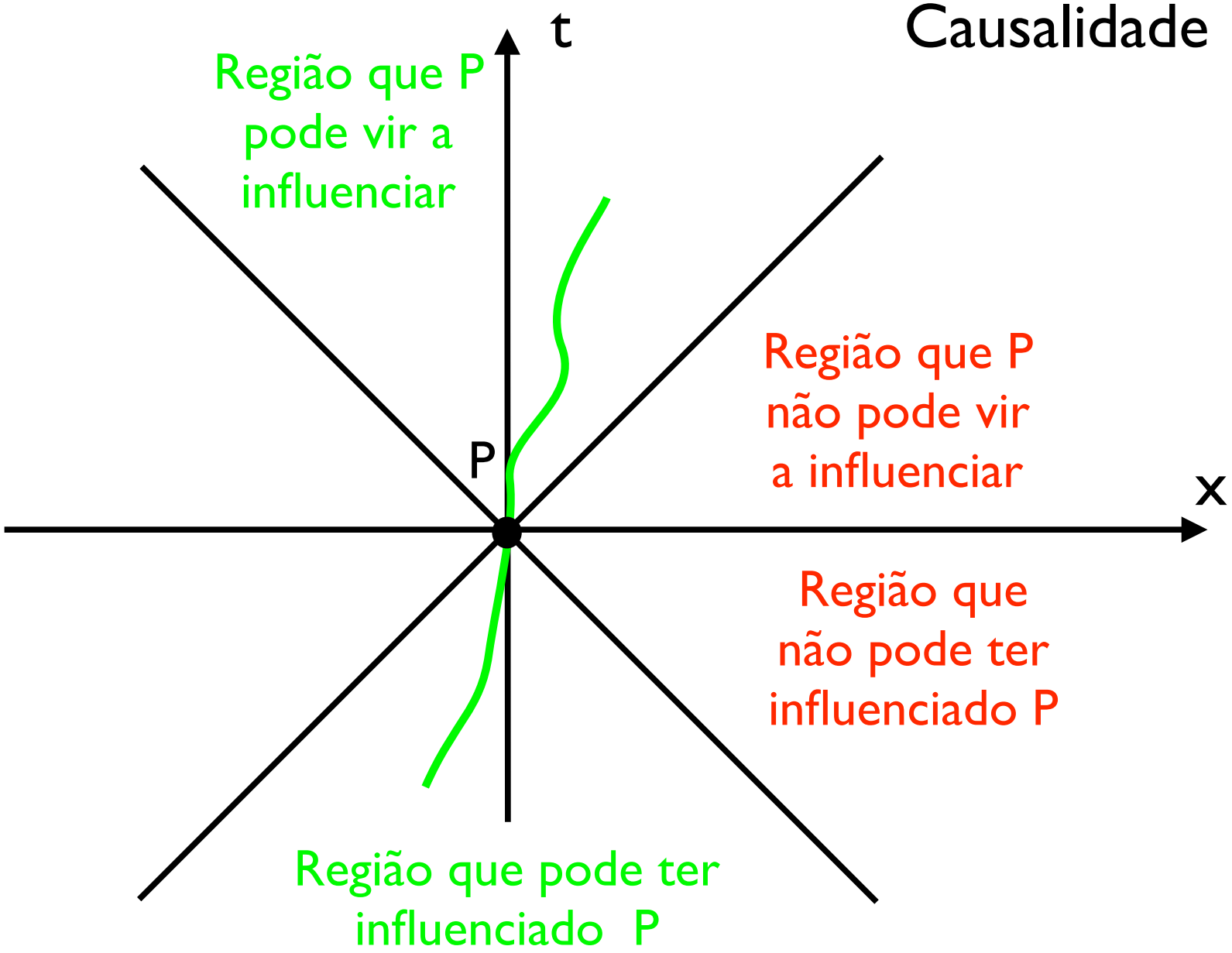
Cone de Luz

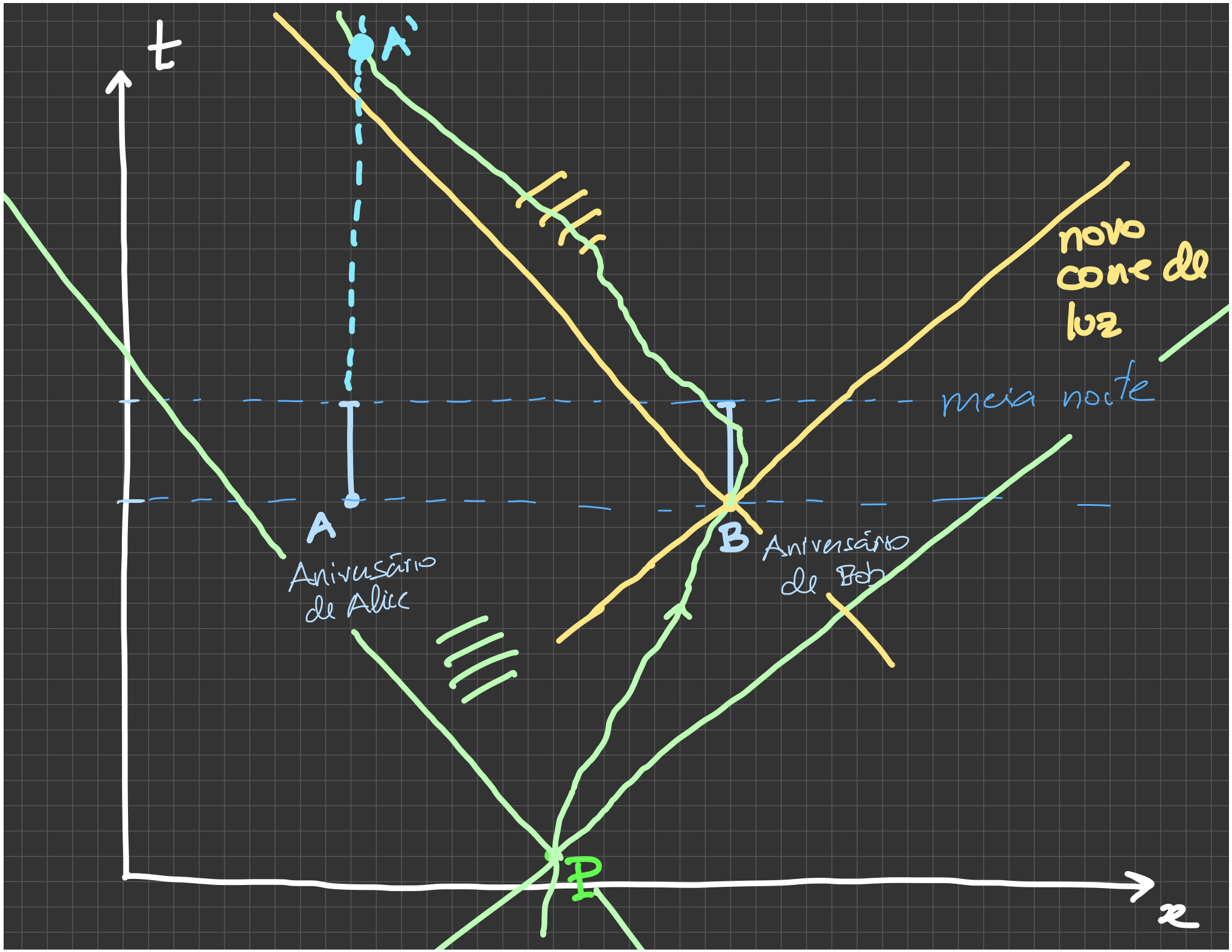

t
1 seg
luz a ir p/ direita
45°
Sempre assim!
1 s
luz
300 000 000 m
600 000 000 m
escala
x
y

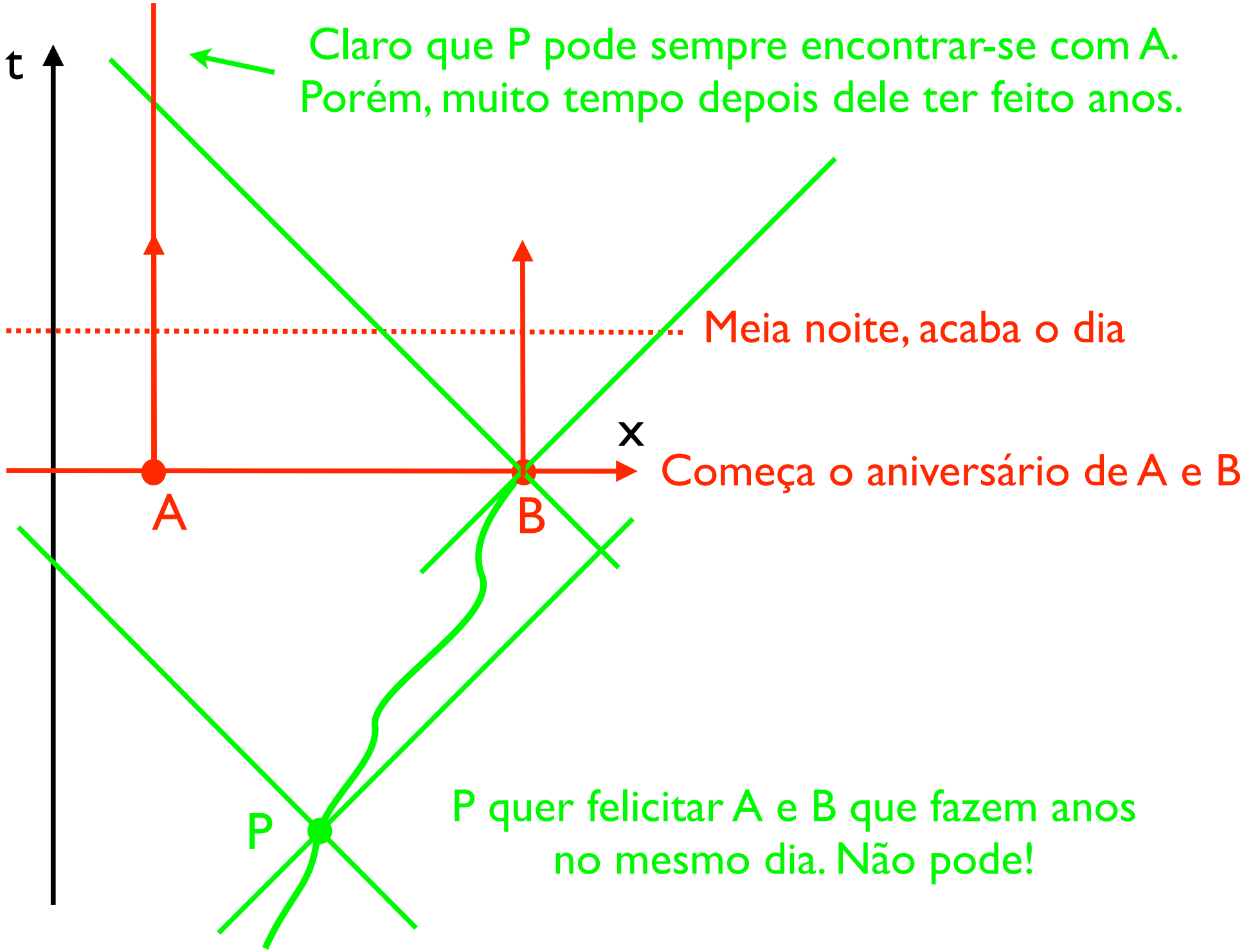

600.000.000 metros
Não Ok
Ok
1 segundo
c=300.000.000 m/seg

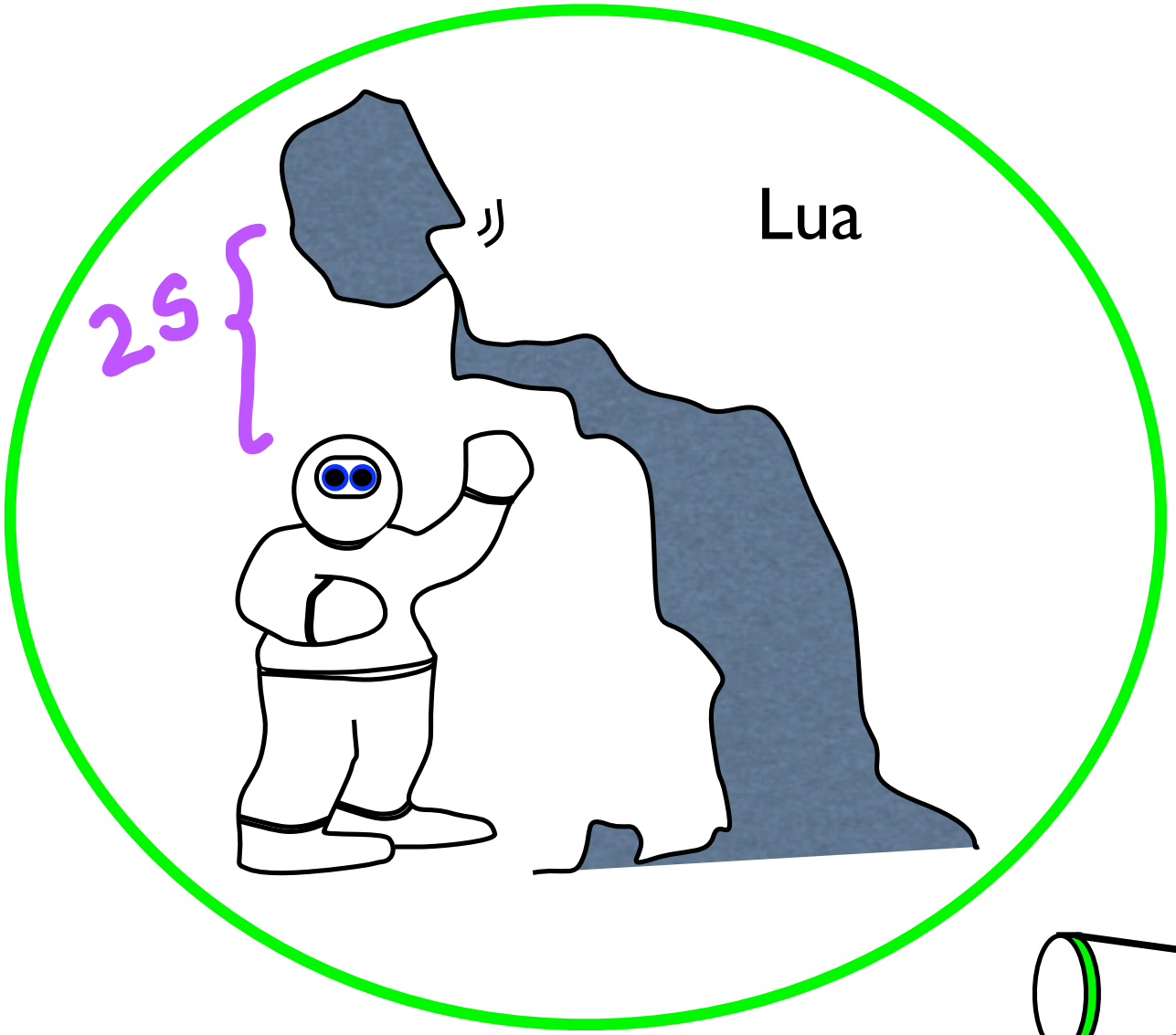

Região que P pode influenciar
1
2
P
Região que P não pode influenciar
Região q P não pode influ. P.
Região que pode ter influenciado P

Causalidade



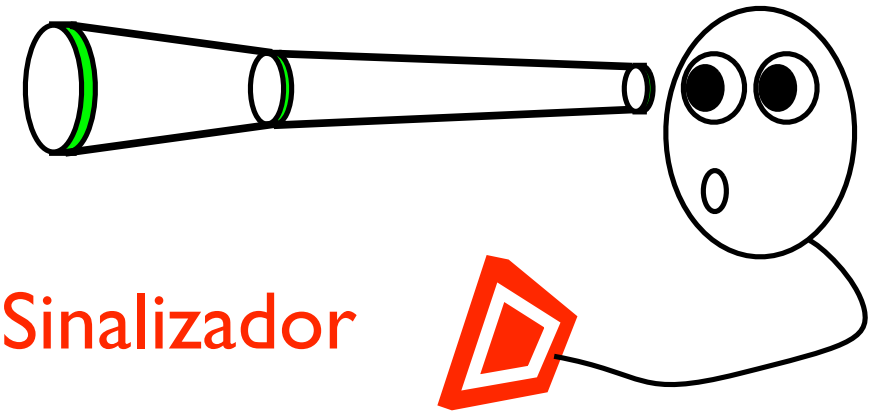

t
A'
A
B
P
meia noite
novo cone de luz
Aniversário de Alice
Aniversário de Bob
x


t
x
A
B
P
Claro que P pode sempre encontrar-se com A.
Porém, muito tempo depois dele ter feito anos.
Meia noite, acaba o dia
Começa o aniversário de A e B
P quer felicitar A e B que fazem anos
no mesmo dia. Não pode!

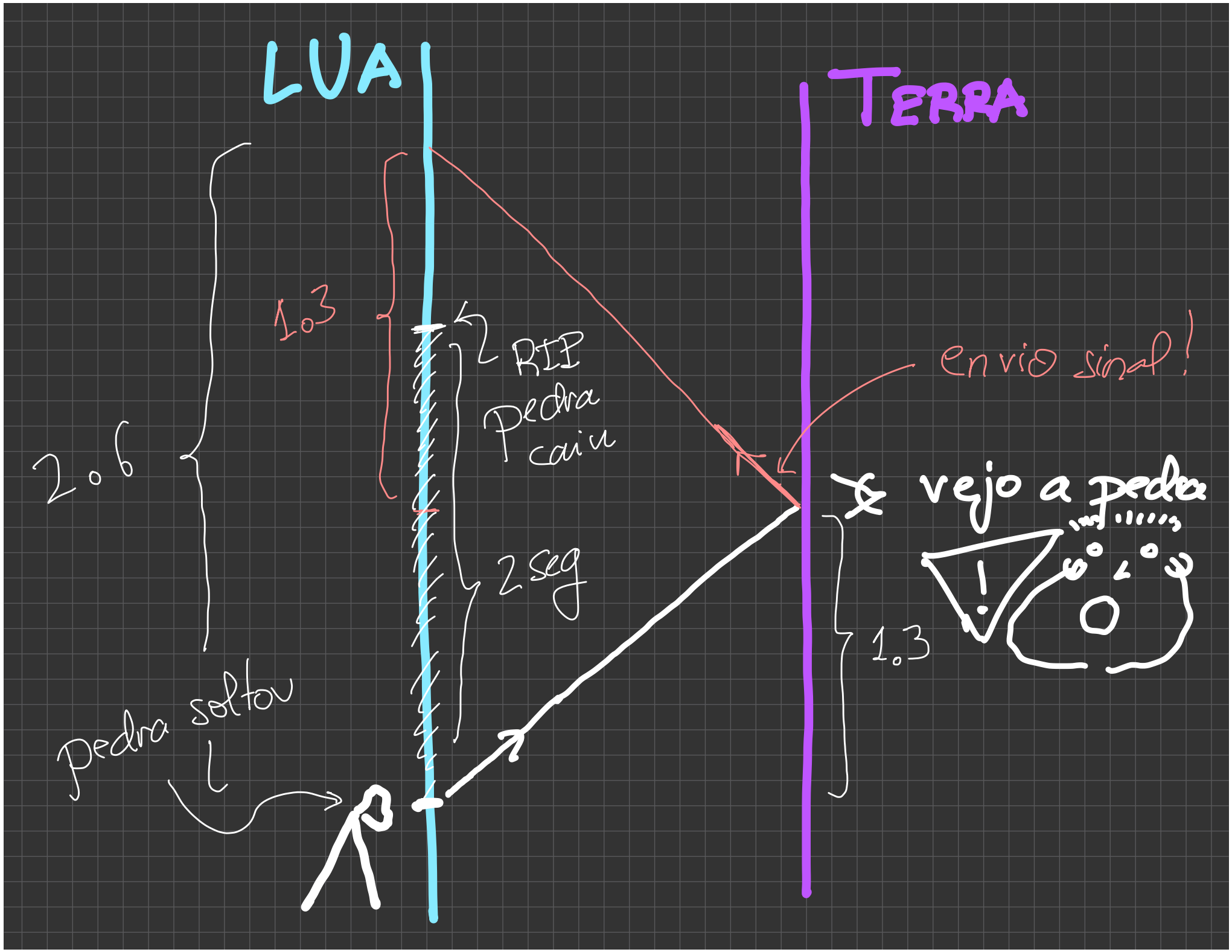


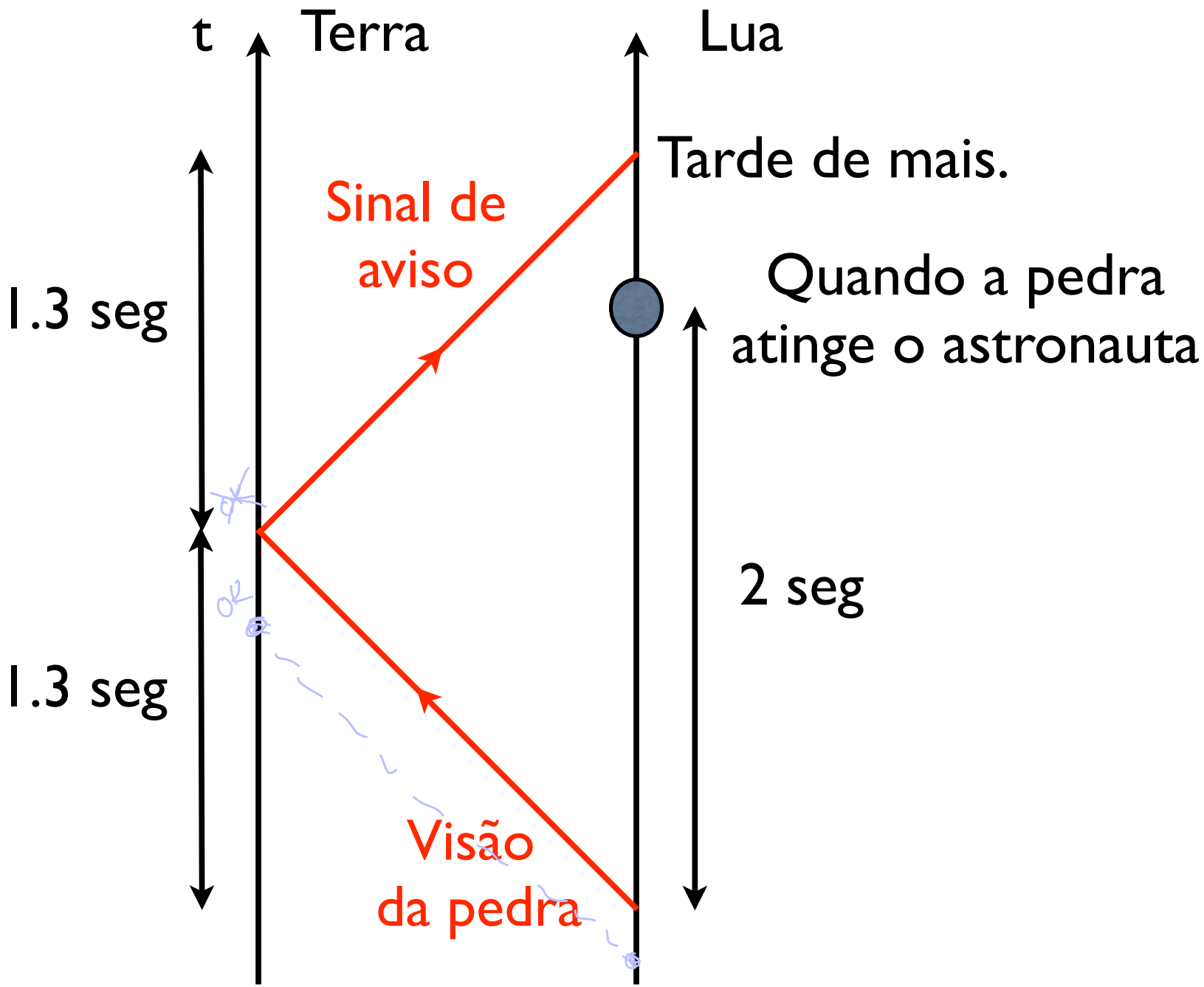
Terra

A pedra demorará 2 segundos a cair-lhe em cima mas como eu vejo o passado...

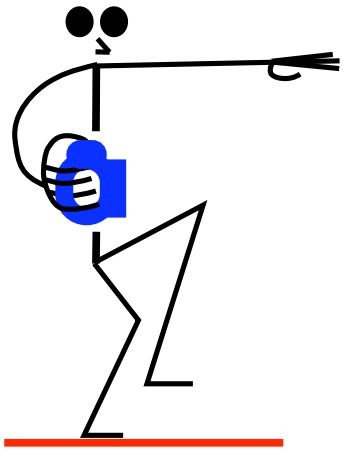


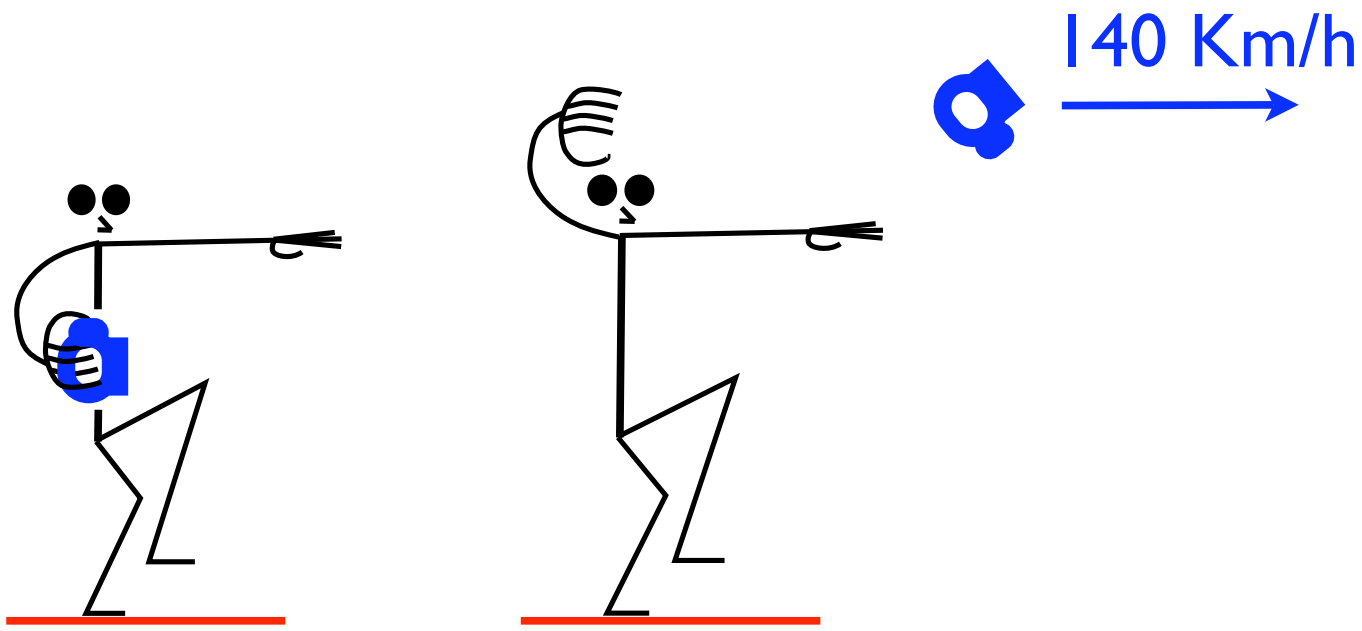
Sinalizador

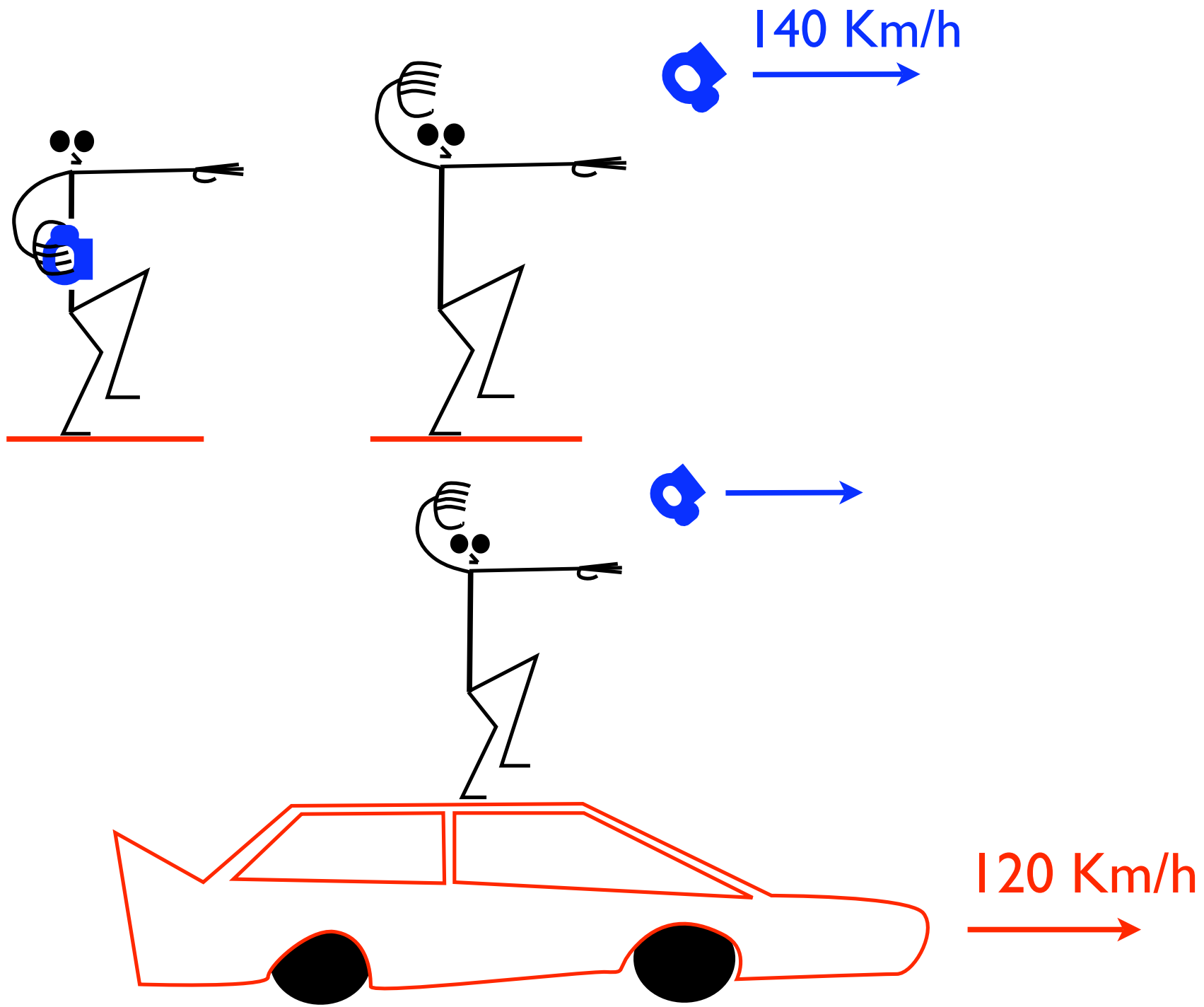

LUA
TERRA
1.3
2.6
RIP
Pedra
caiu
2 seg
envio sinal!
vejo a pedra
1.3
pedra soltou

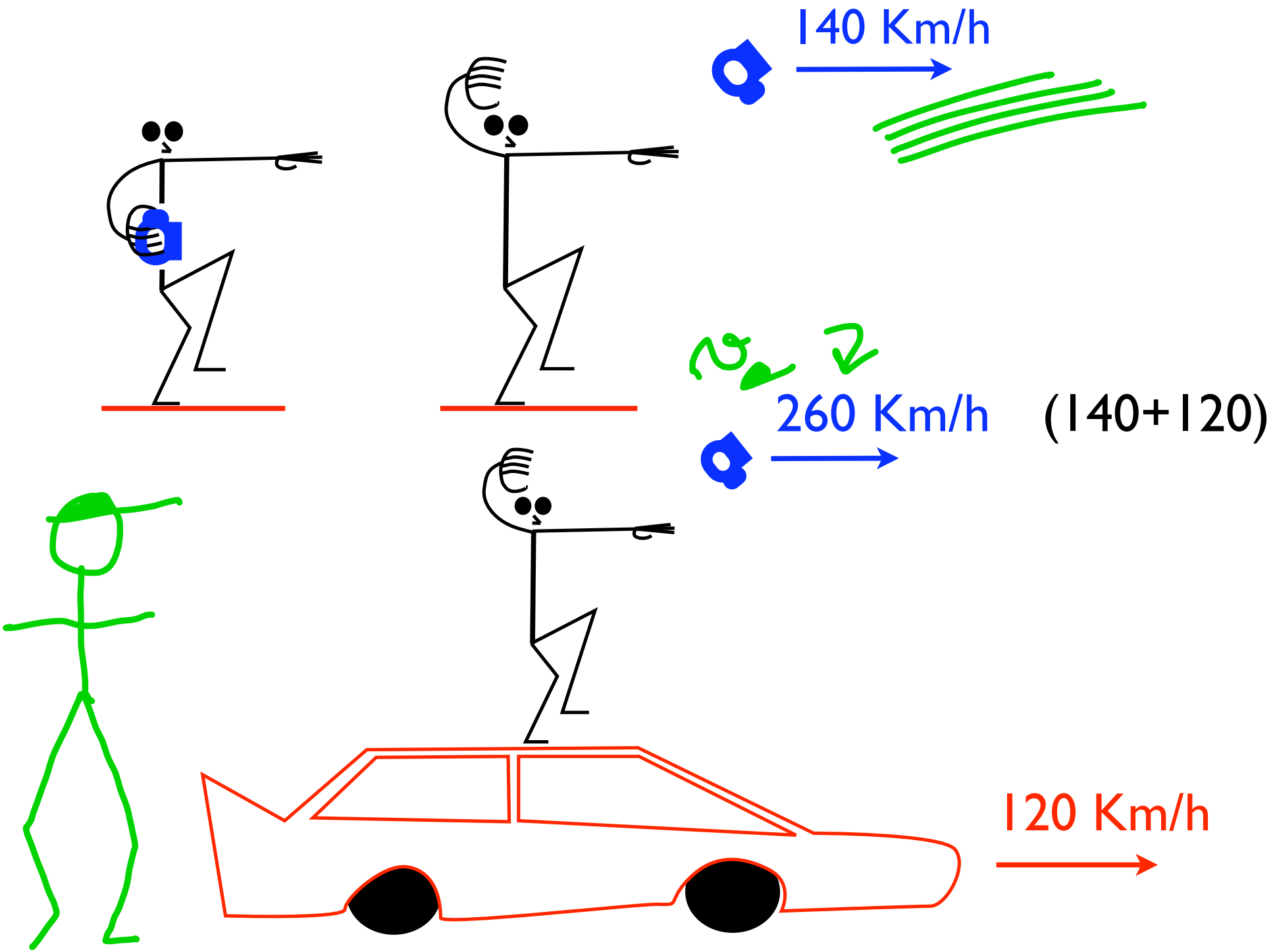

t
Terra
Lua
Tarde de mais.
Sinal de
aviso
1.3 seg
Quando a pedra
atinge o astronauta
2 seg
1.3 seg
Visão
da pedra

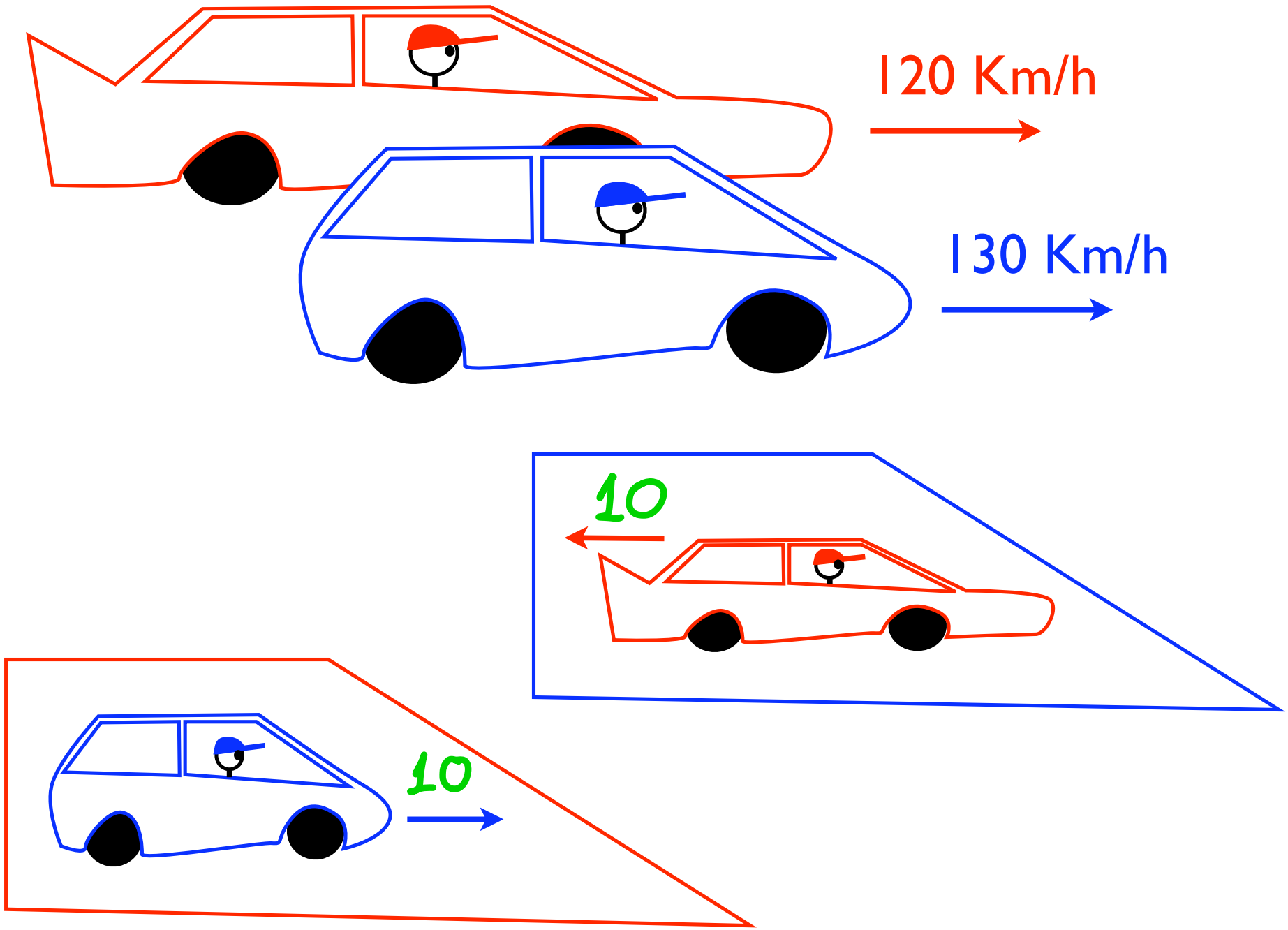
Adição de velocidades

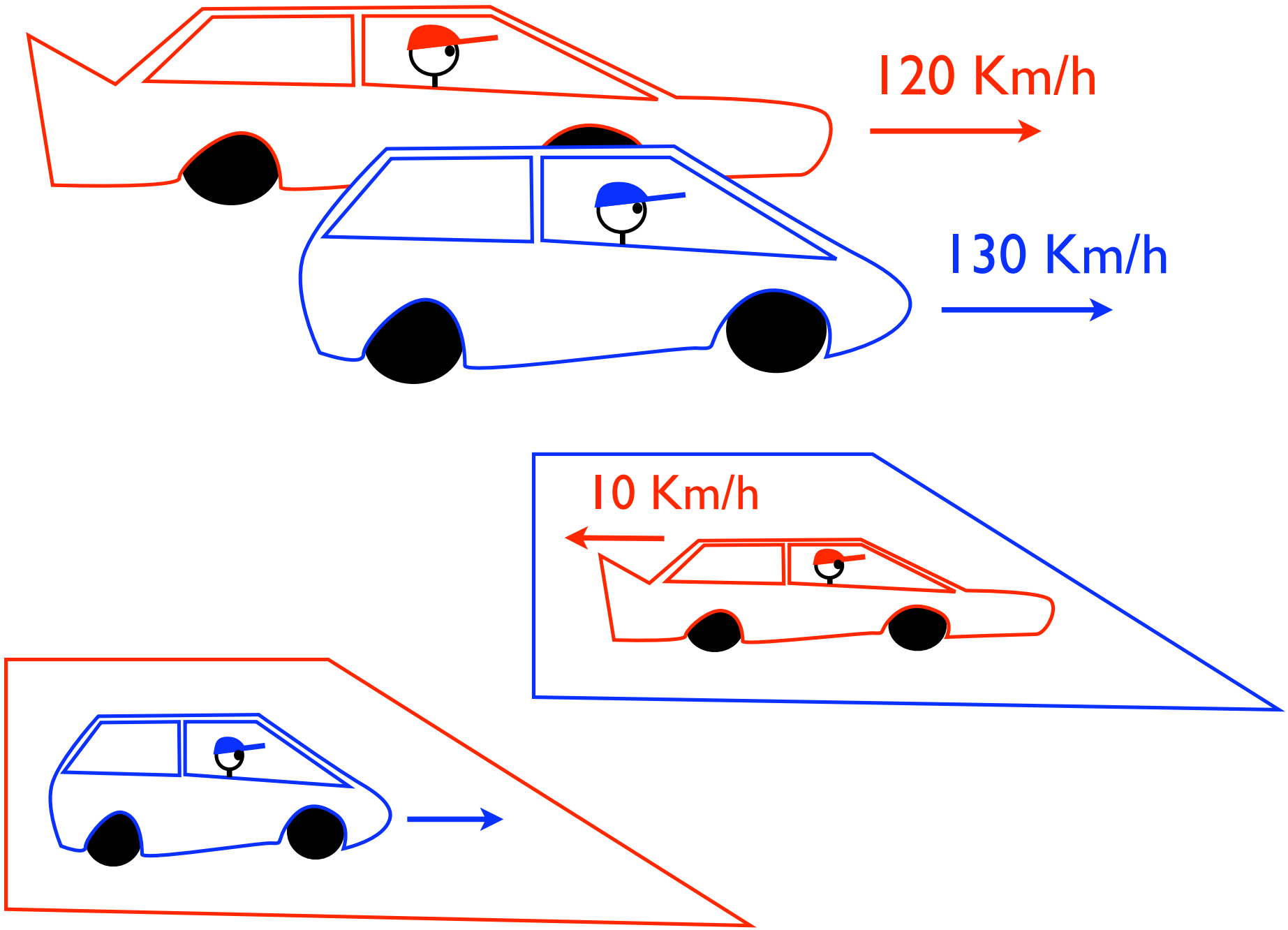


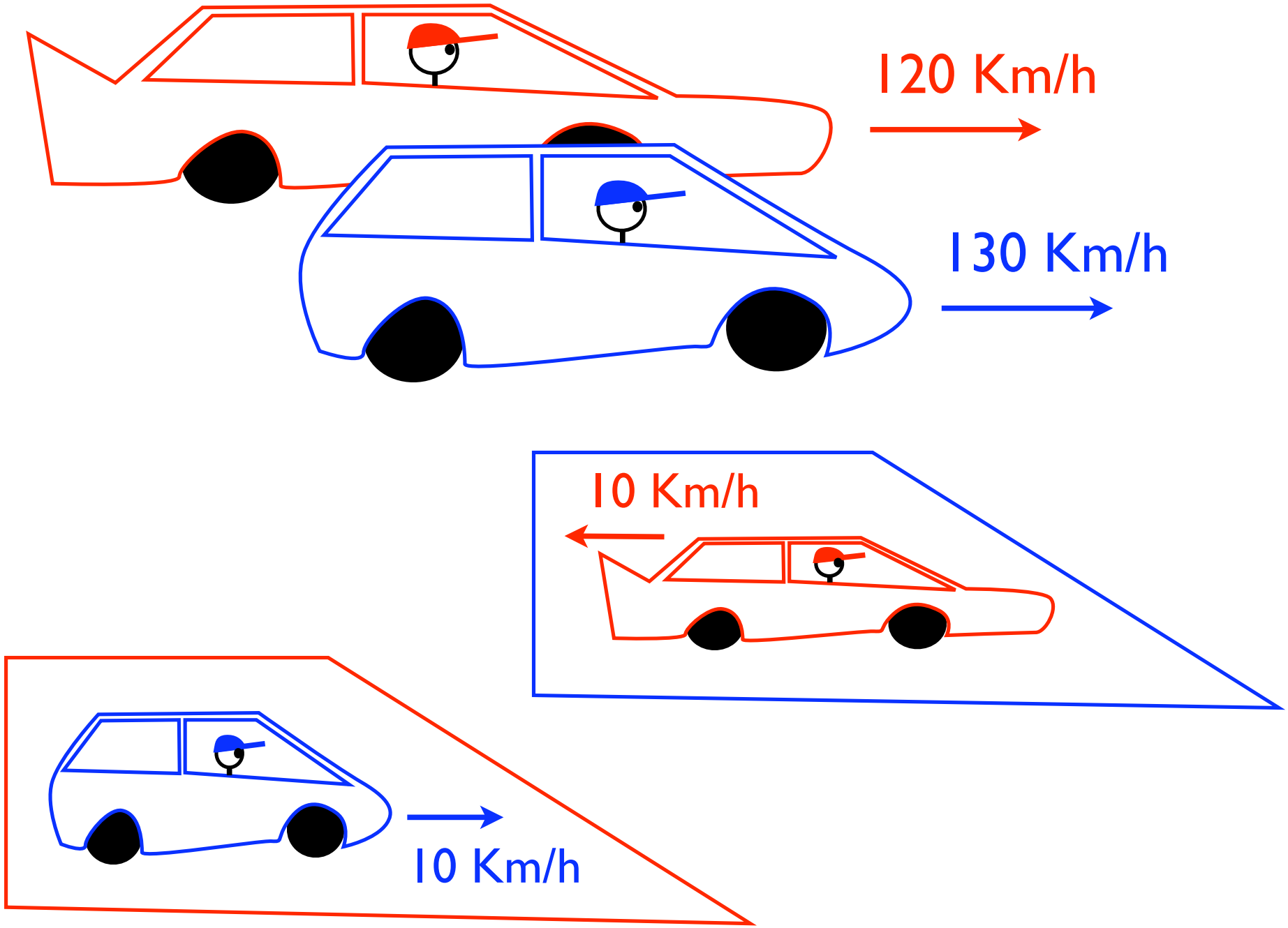

140 Km/h

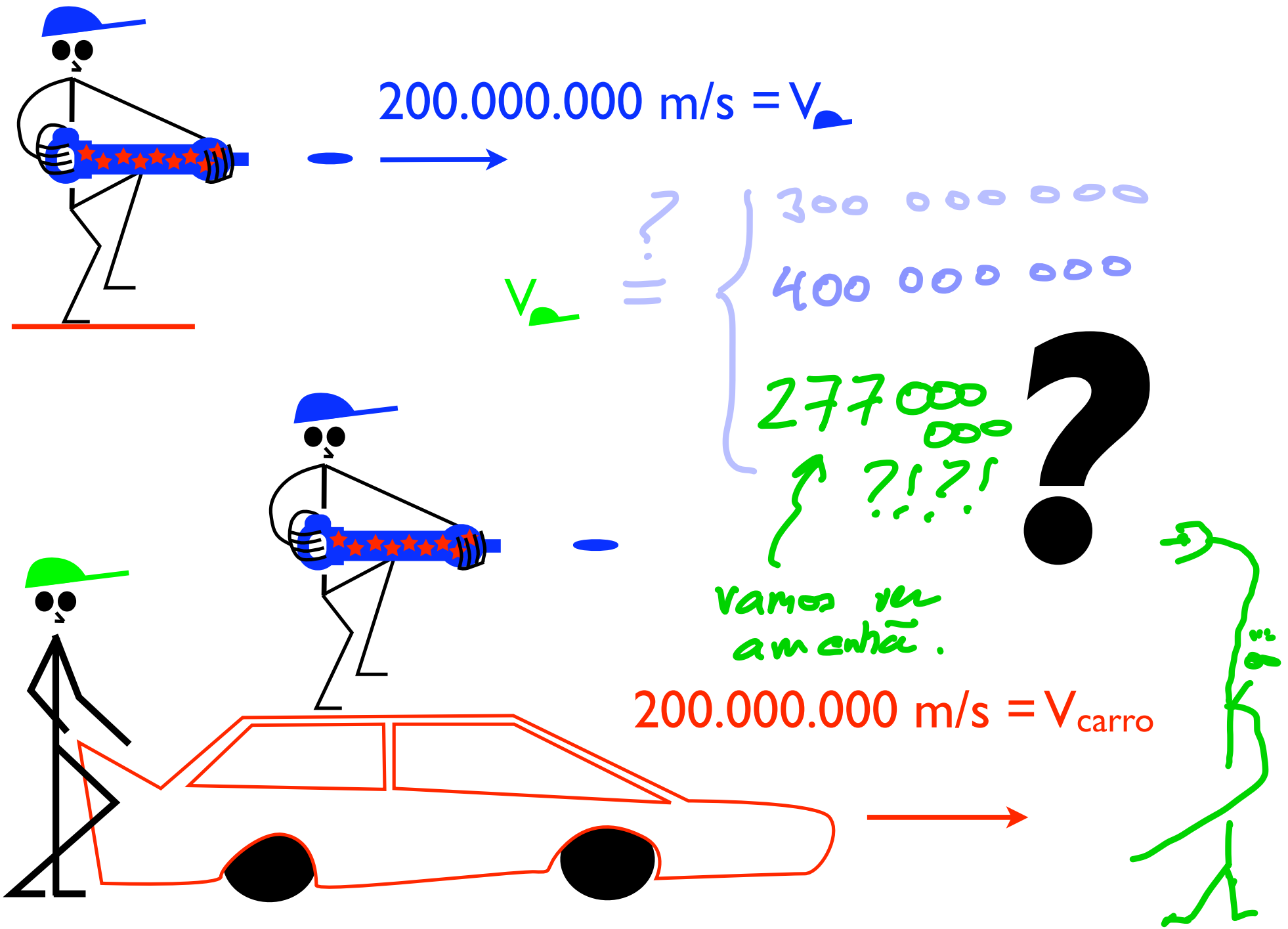

140 Km/h
120 Km/h

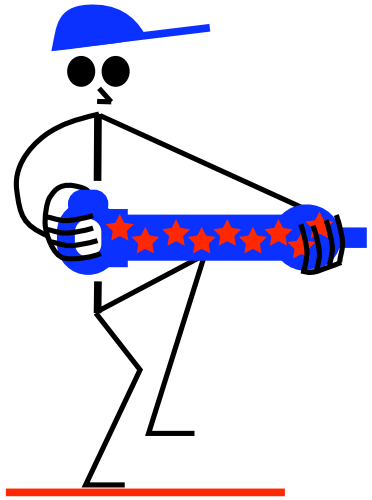

140 Km/h
260 Km/h
(140+120)
120 Km/h


120 Km/h
130 Km/h
10
10


120 Km/h
130 Km/h
10 Km/h


120 Km/h
130 Km/h
10 Km/h
10 Km/h


200.000.000 m/s = V
V = ?
300 000 000
400 000 000
277 000 000
?!?!
?
vamos ver amanhã.
200.000.000 m/s = V carro



200.000.000 m/s = V_{boy}

V_{boy} = 200.000.000 + 200.000.000

= V_{boy} + V_{carro}

= 400.000.000 m/s

?

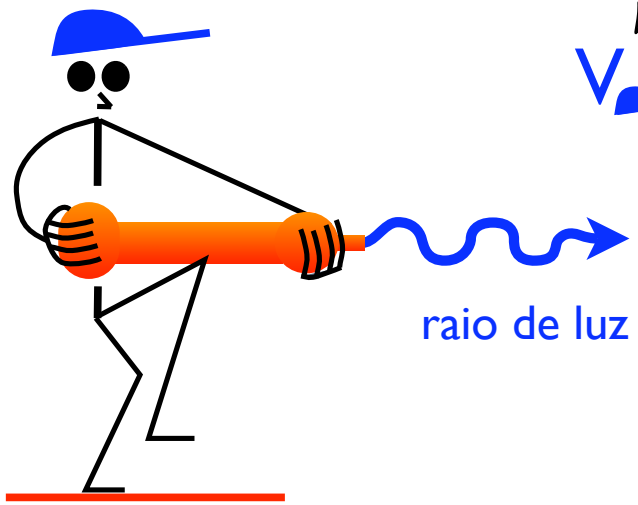
200.000.000 m/s = V_{carro}

Não!

400.000.000 m/s > 300.000.000 m/s = c

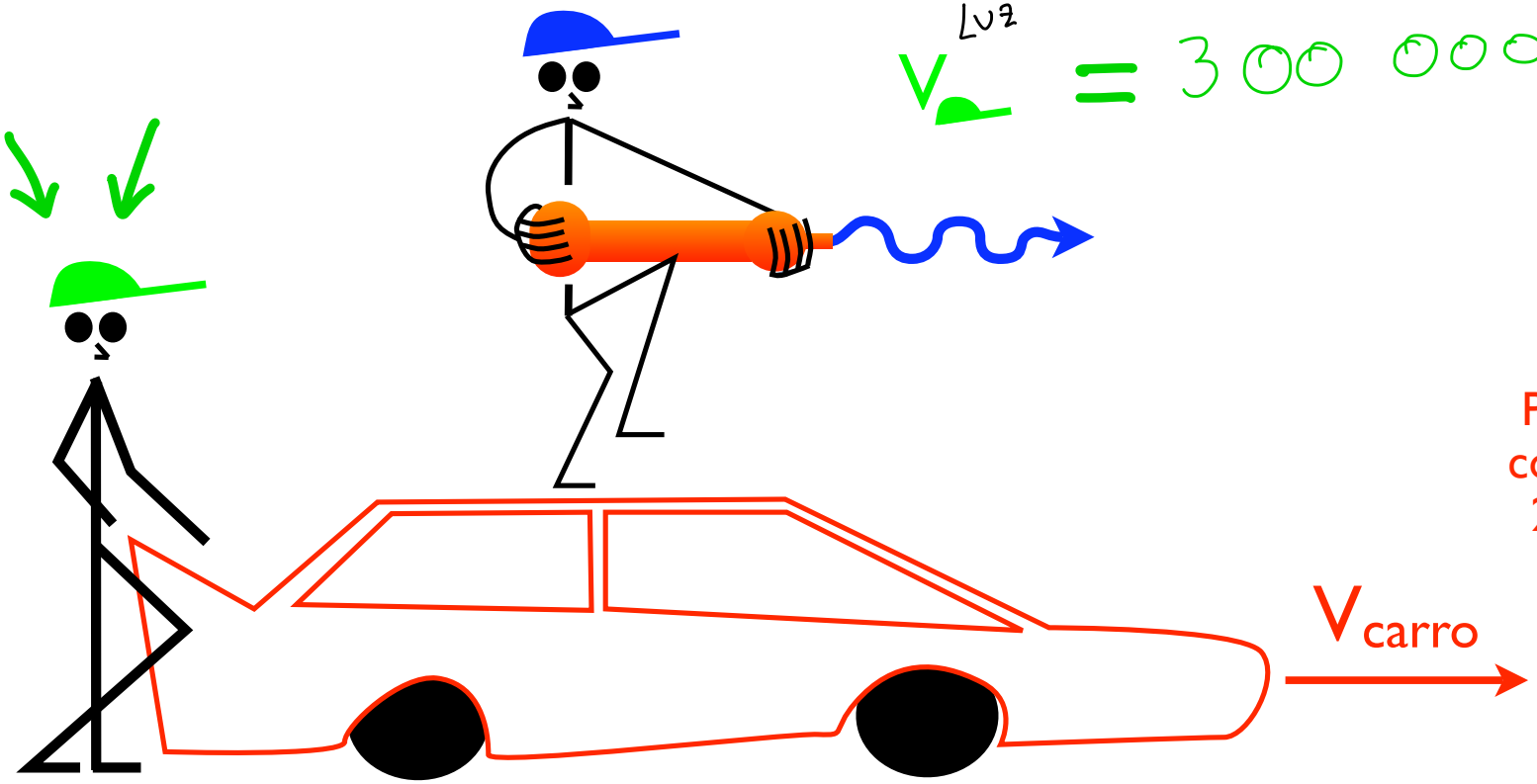
O cálculo que parecia normal tem um erro de pelo menos 100.000.000 m/s !

O Postulado



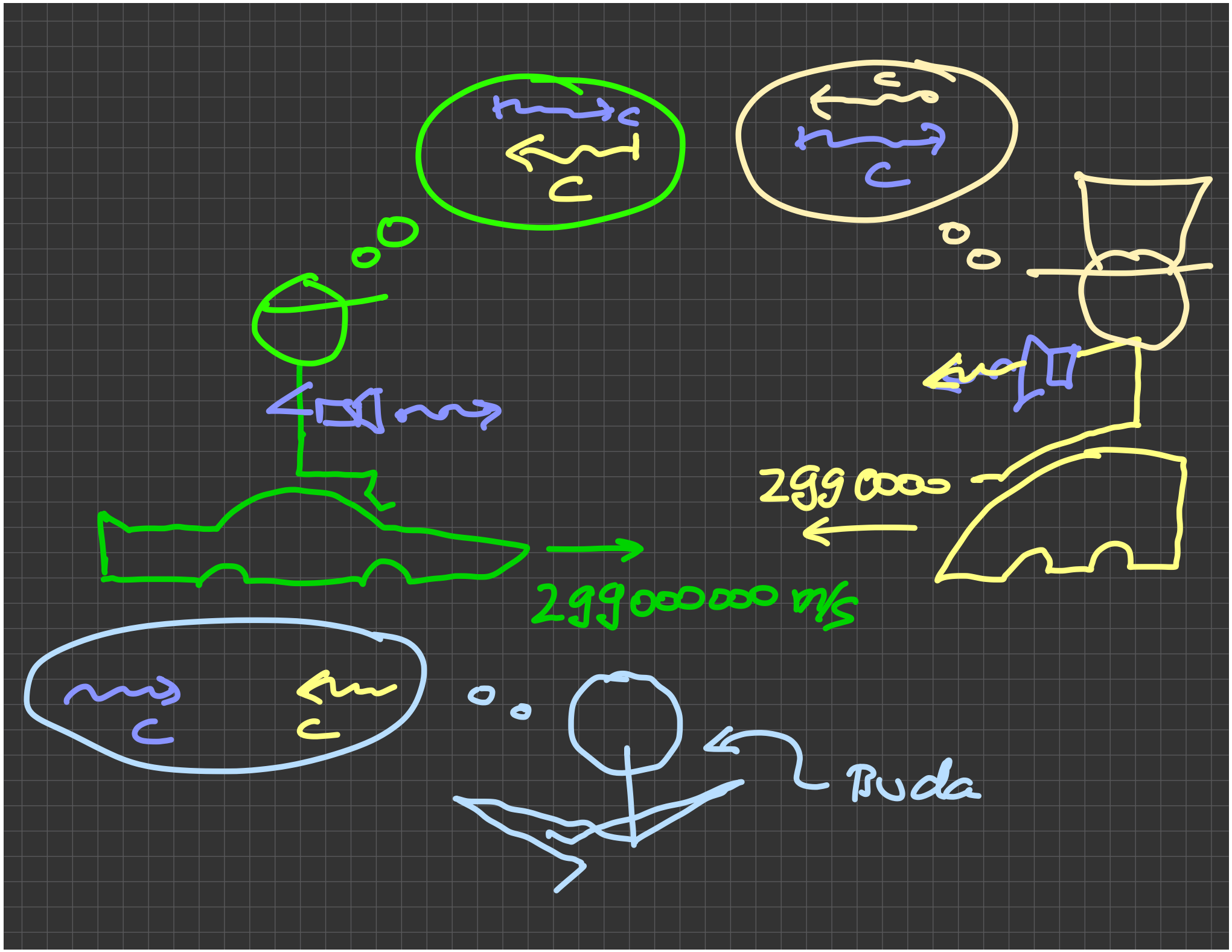
$V_{luz} = 300.000.000 \text{ m/s} = c$

Sempre c



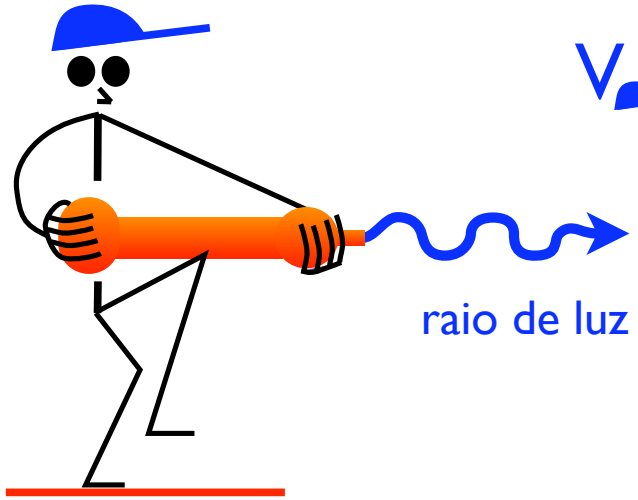
$V_{luz} = 300\,000\,000 = c$

Pode ser qualquer coisa, (por exemplo 200.000.000 m/s)


c
c
c
c
299000
299000000 m/s
Buda

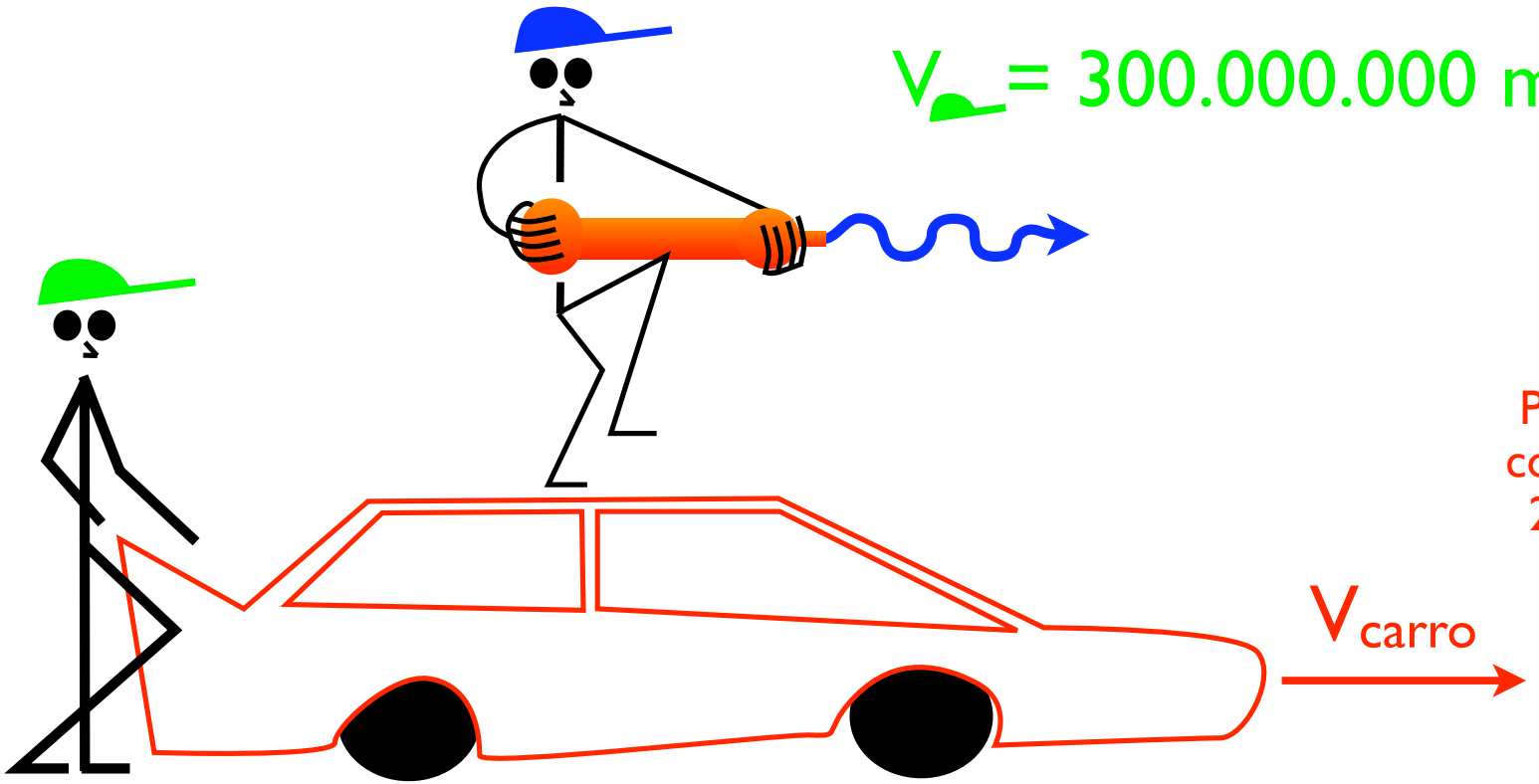
Postulado : Invariância da c

Consequências : Relatividade de
- tempo (dil.)
- espaço (cont.)
- simult.
- .....



V_{luz} = 300.000.000 m/s = c

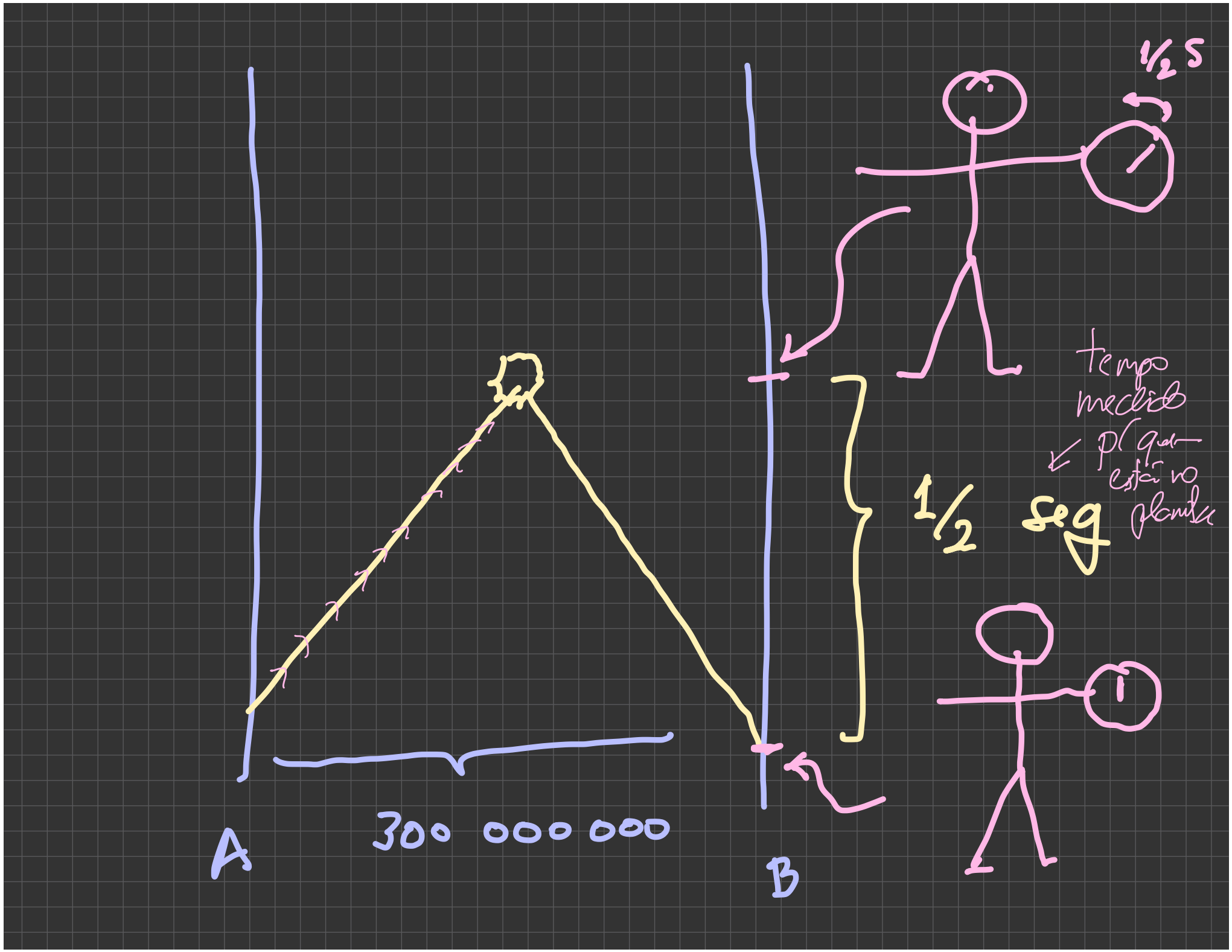
Independentemente da velocidade do carro



V_{luz} = 300.000.000 m/s = c

Pode ser qualquer coisa, (por exemplo 200.000.000 m/s)

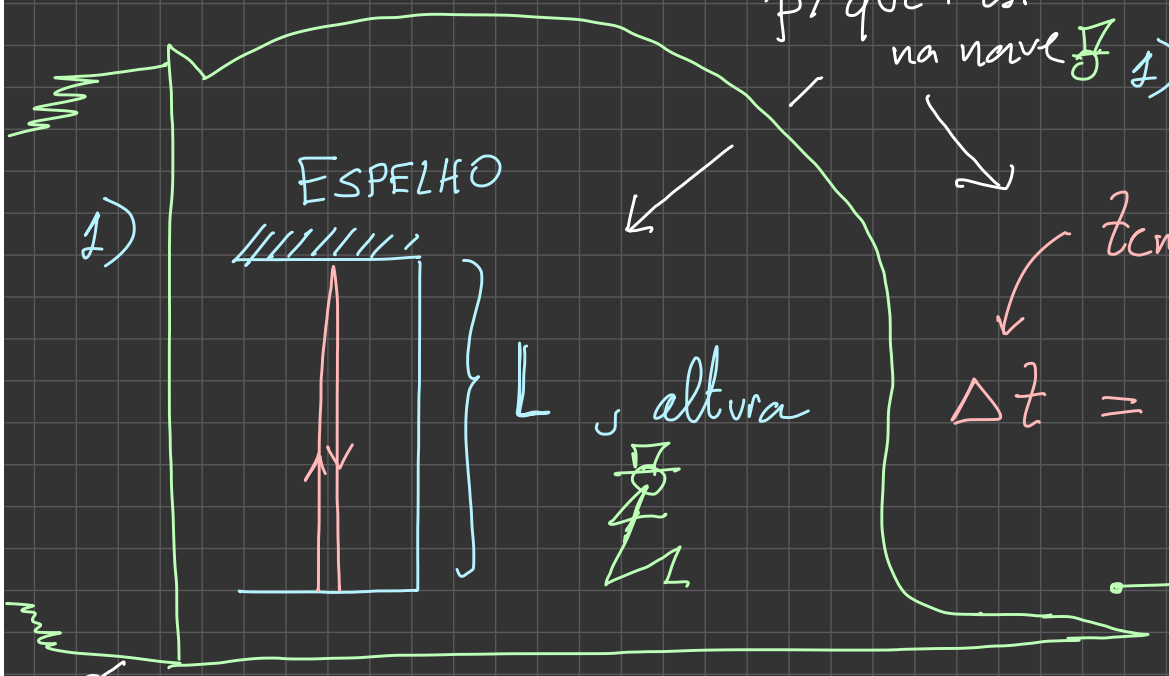
V_{carro}


1/2 s
tempo medido por quem está no planeta
1/2 seg
A
300 000 000
B

Dilatação do Tempo

(amanhã)

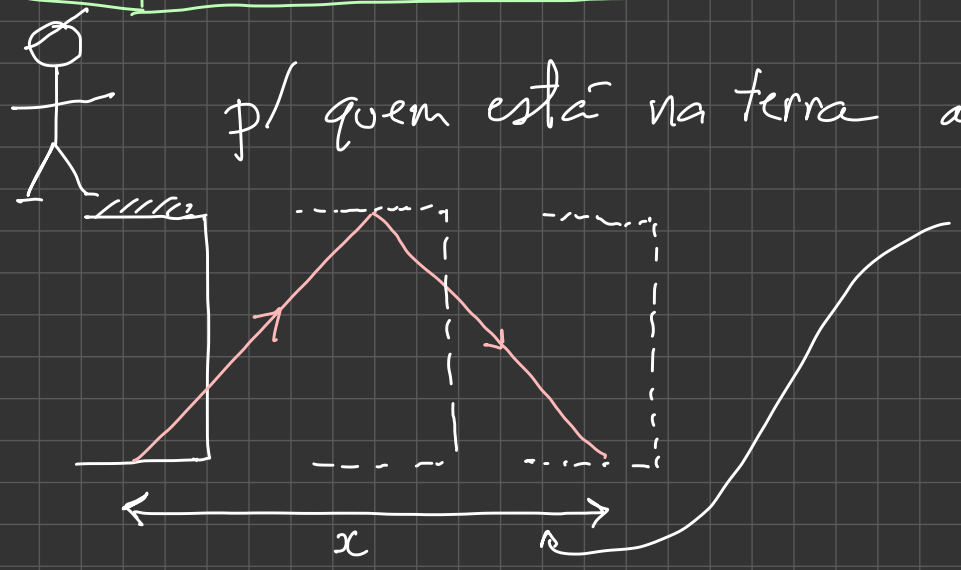
c é igual p/ todos ⟹ tempo não é igual p/ todos



tempo p/ luz subir e descer

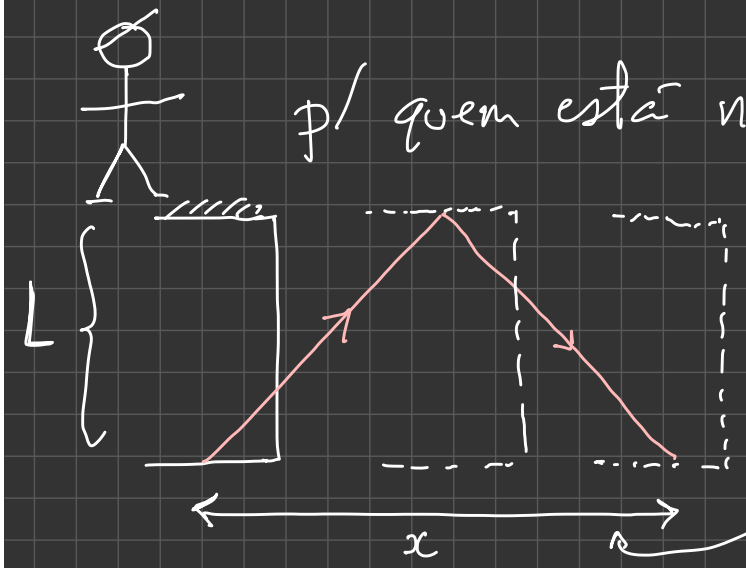
$$\Delta t = \frac{2L}{c}$$

($t_{total} = n \times \Delta t$)
"só contar"



p/ quem está na terra a luz demora $\Delta t'$
o que a nave anda enquanto a luz sobe e desce.

$$x = \Delta t' \times v$$



p/ quem está na terra a luz demora $\Delta t'$ o que a nave anda enquanto a luz sobe e desce.

$$x = \Delta t' \times v$$

na nave

$$\Delta t = \frac{2L}{c}$$

$$(d_{subida})^2 = \left(\frac{x}{2}\right)^2 + L^2$$

$$d = 2\sqrt{\left(\frac{x}{2}\right)^2 + L^2}$$

contas

na terra

$$\Delta t' = \frac{d}{c}$$

resultado p/

$$\Delta t \longleftrightarrow \Delta t'$$

$$x = \Delta t' \times v \quad (1)$$

$$\Delta t = \frac{2L}{c} \quad (2)$$

$$d = 2\sqrt{\left(\frac{x}{2}\right)^2 + L^2} \quad (3)$$

$$\Delta t' = \frac{d}{c} \quad (4)$$

$$\Delta t \overset{(2)}{=} \frac{2}{c} L \overset{(3)}{=} \frac{2}{c}\sqrt{\left(\frac{d}{2}\right)^2 - \left(\frac{x}{2}\right)^2} \overset{(4)}{=}$$

$$= \frac{2}{c}\sqrt{\frac{c^2(\Delta t')^2}{4} - \frac{x^2}{4}} \overset{(1)}{=} \frac{2}{c}\sqrt{\frac{c^2 \Delta t'^2}{4} - \frac{v^2 \Delta t'^2}{4}}$$

$$\Delta t' \sqrt{1 - \frac{v^2}{c^2}}$$

Dilatação do tempo:

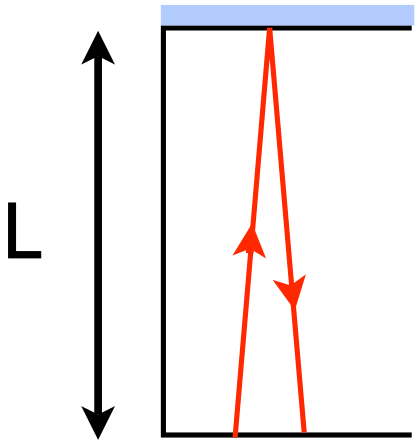
$$\Delta t = \Delta t' \sqrt{1 - \frac{v^2}{c^2}}$$

- v: velocidade da nave
- c: velocidade da luz
- $\Delta t'$: tempo medido por quem está na terra
- Δt: tempo p/ quem está na nave

TEMPO P/ QUEM ESTÁ NA NAVE

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

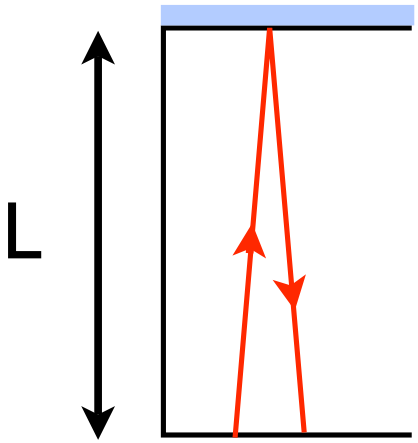
$\sqrt{1 - \frac{v^2}{c^2}} < 1$, logo $\Delta t' > \Delta t$



Relógio de luz

$$t = \frac{2L}{c}$$

velocidade = $\frac{\text{distância}}{\text{tempo}}$



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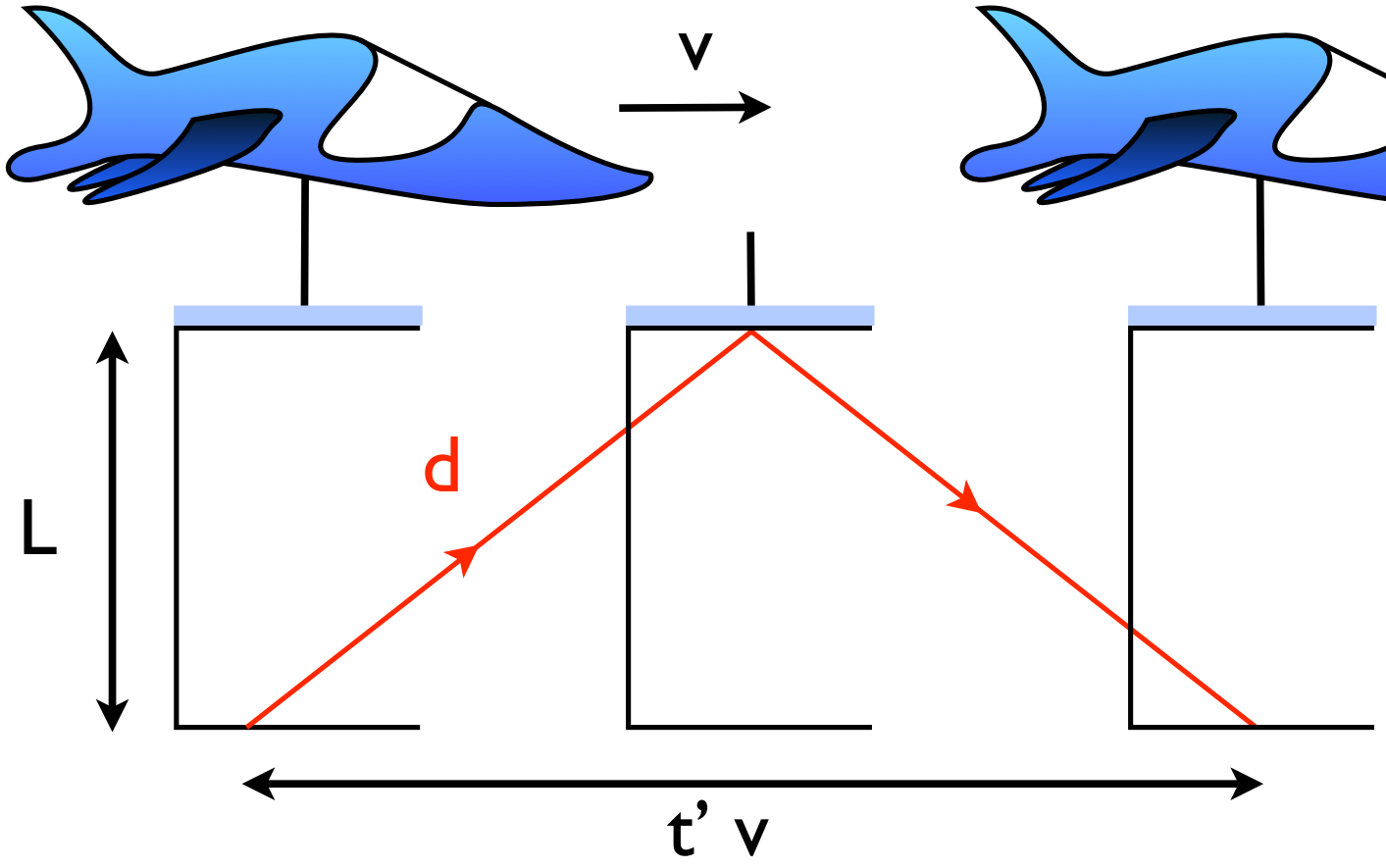
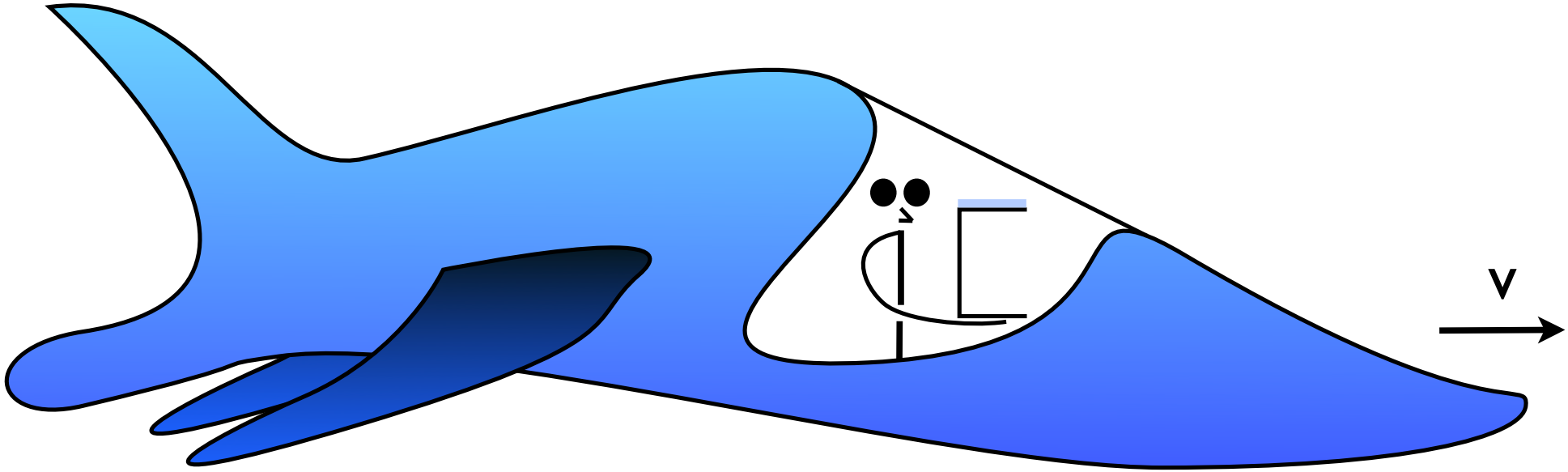


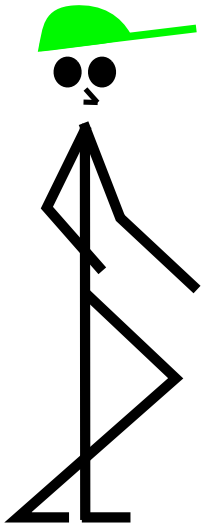
$$t' = \frac{2d}{c}$$

$$d^2 = L^2 + \left(\frac{t'v}{2}\right)^2$$

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

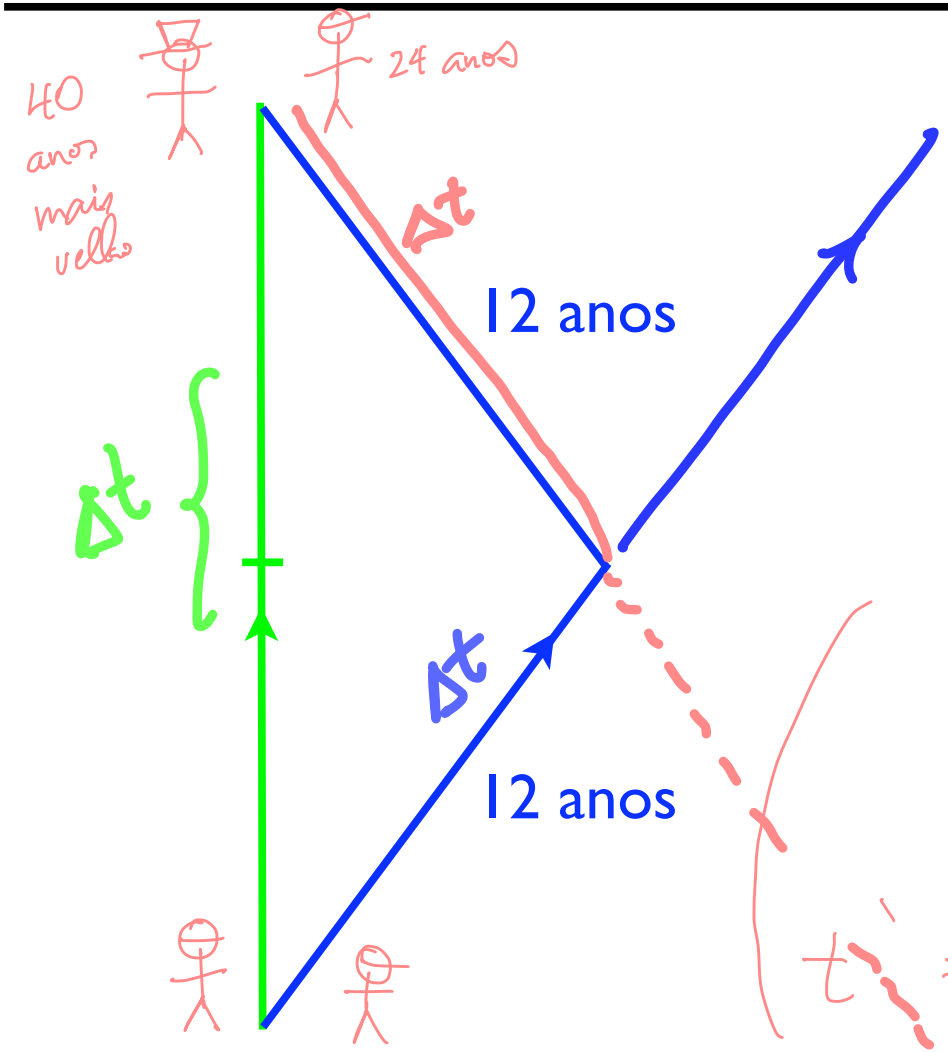


t' , o tempo medido por alguém em **terra**,
é **maior** do que
t , o tempo medido por alguém na **nave**



$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Se eu, na nave, a 240.000.000 m/s (0.8c), jogar um jogo de xadrez de 120 minutos, um espectador, na terra, dirá que o jogo demorou 200 minutos.



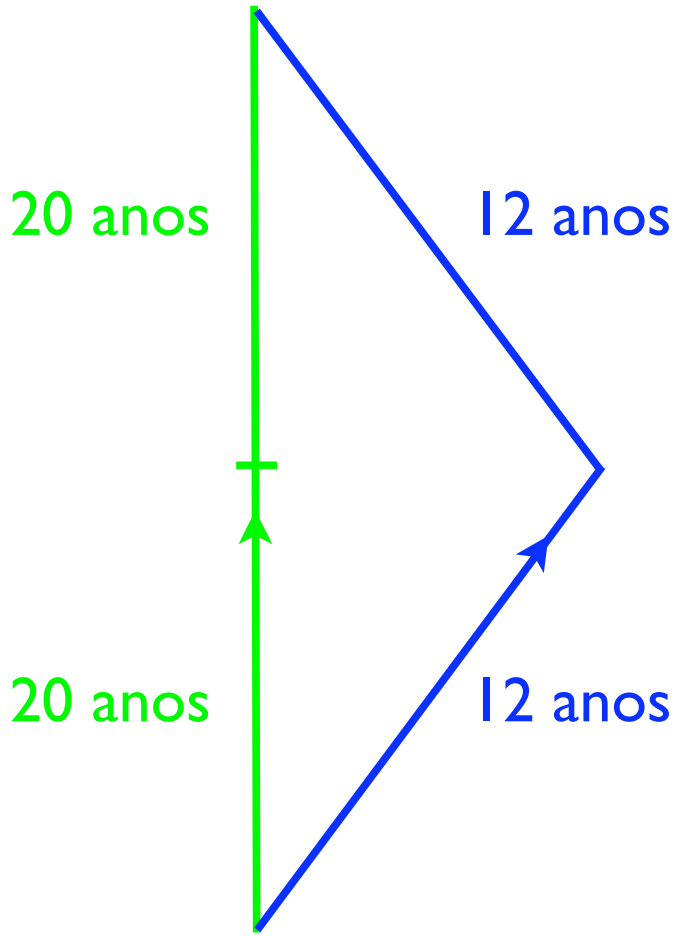
V = 0.8c

12 anos a afastar e
12 anos a aproximar

$$t' = \frac{t}{\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{120\,\text{min}}{\sqrt{1 - 0.8^2}} = 200$$

Se eu, na nave, a 240.000.000 m/s (0.8c), jogar um jogo de xadrez de 120 minutos, um espectador, na terra, dirá que o jogo demorou 200 minutos.

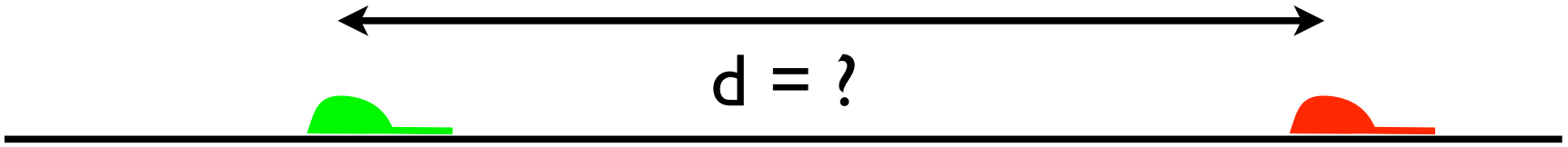


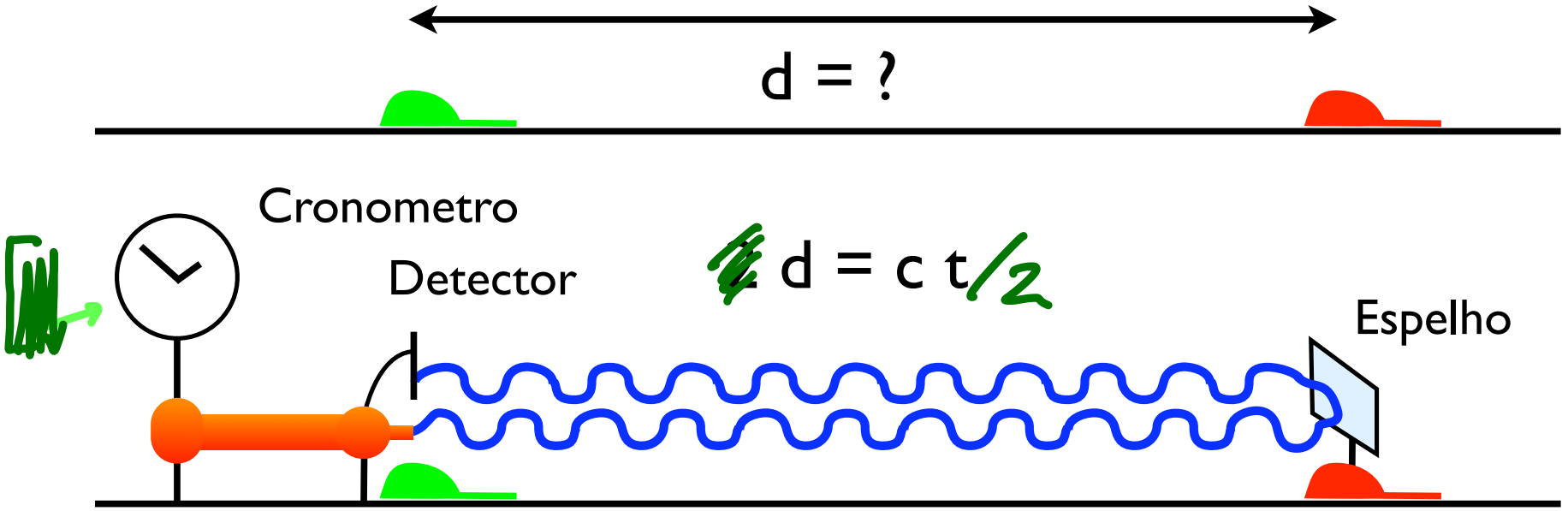
V = 0.8c

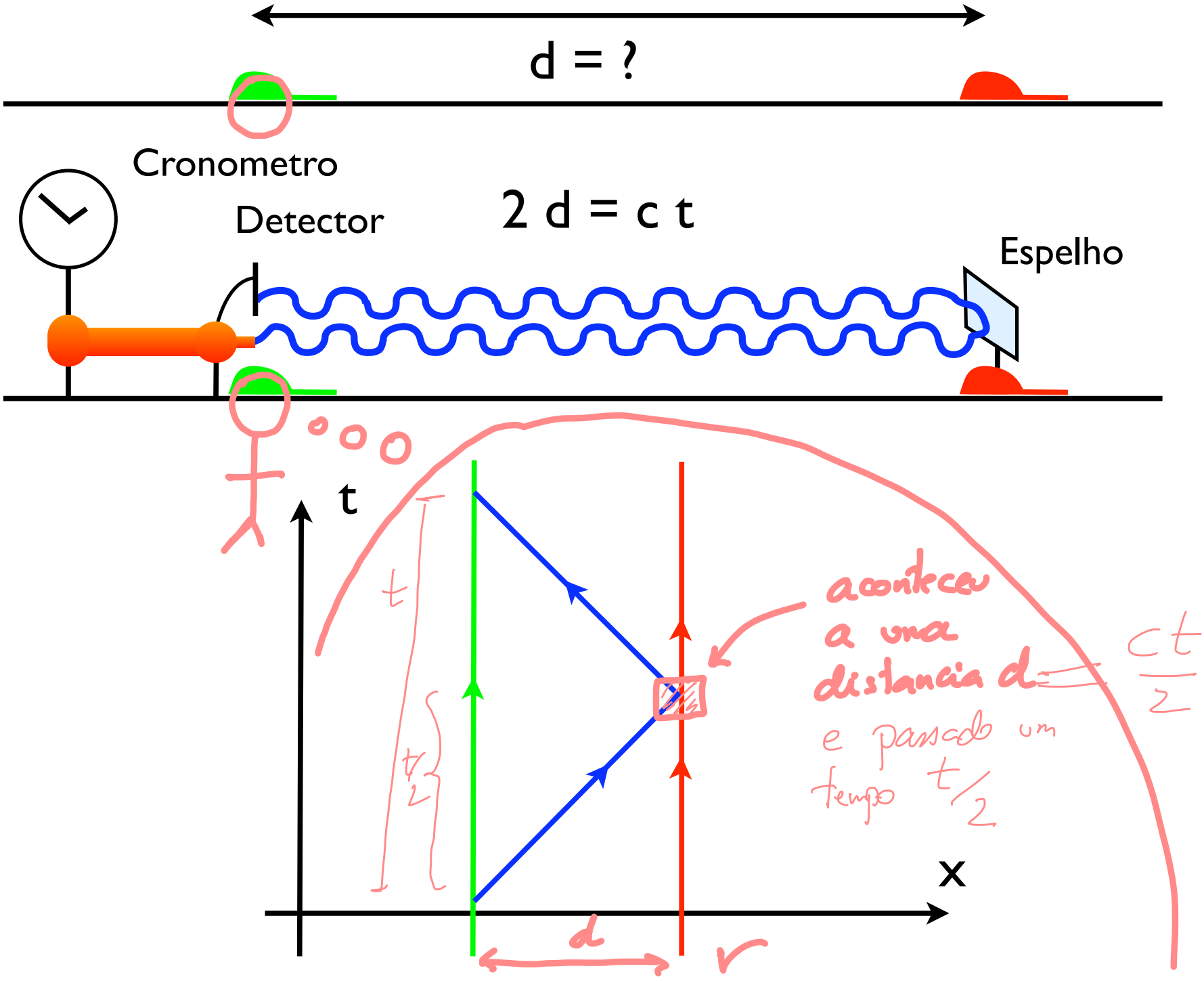
12 anos a afastar e
12 anos a aproximar

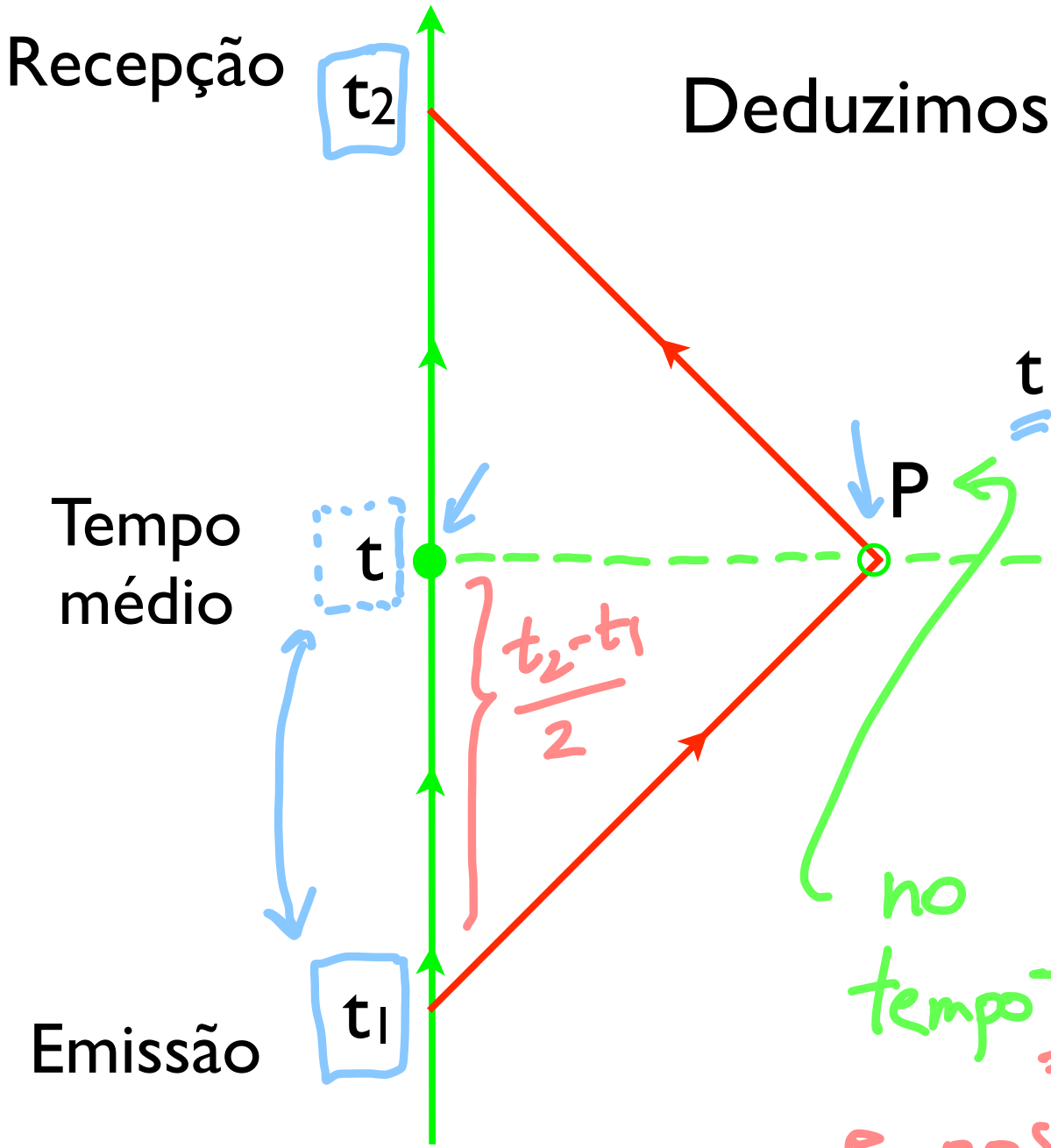
Dois gémeos com 17 anos. Um fica na Terra. Quando o outro regressa tem 41 anos enquanto que o que ficou tem 57!

Distâncias e Simultaneidade


d = ?


d = ?
Cronometro
Detector
d = c t/2
Espelho


d = ?
Cronometro
Detector
2 d = c t
Espelho
t
x
t
t/2
d
acontece a uma distancia d = ct/2 e passado um tempo t/2



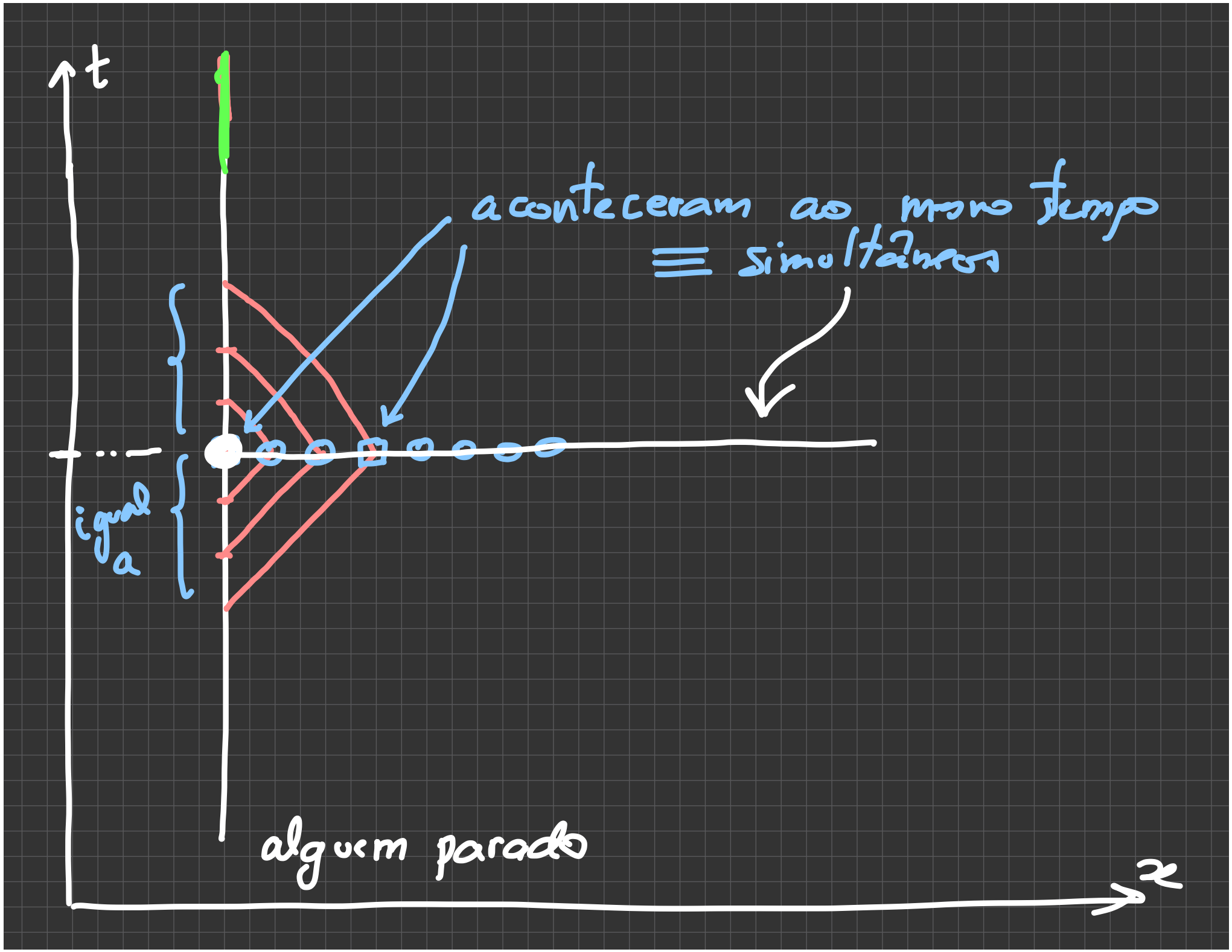
Deduzimos que P ocorreu no instante de tempo

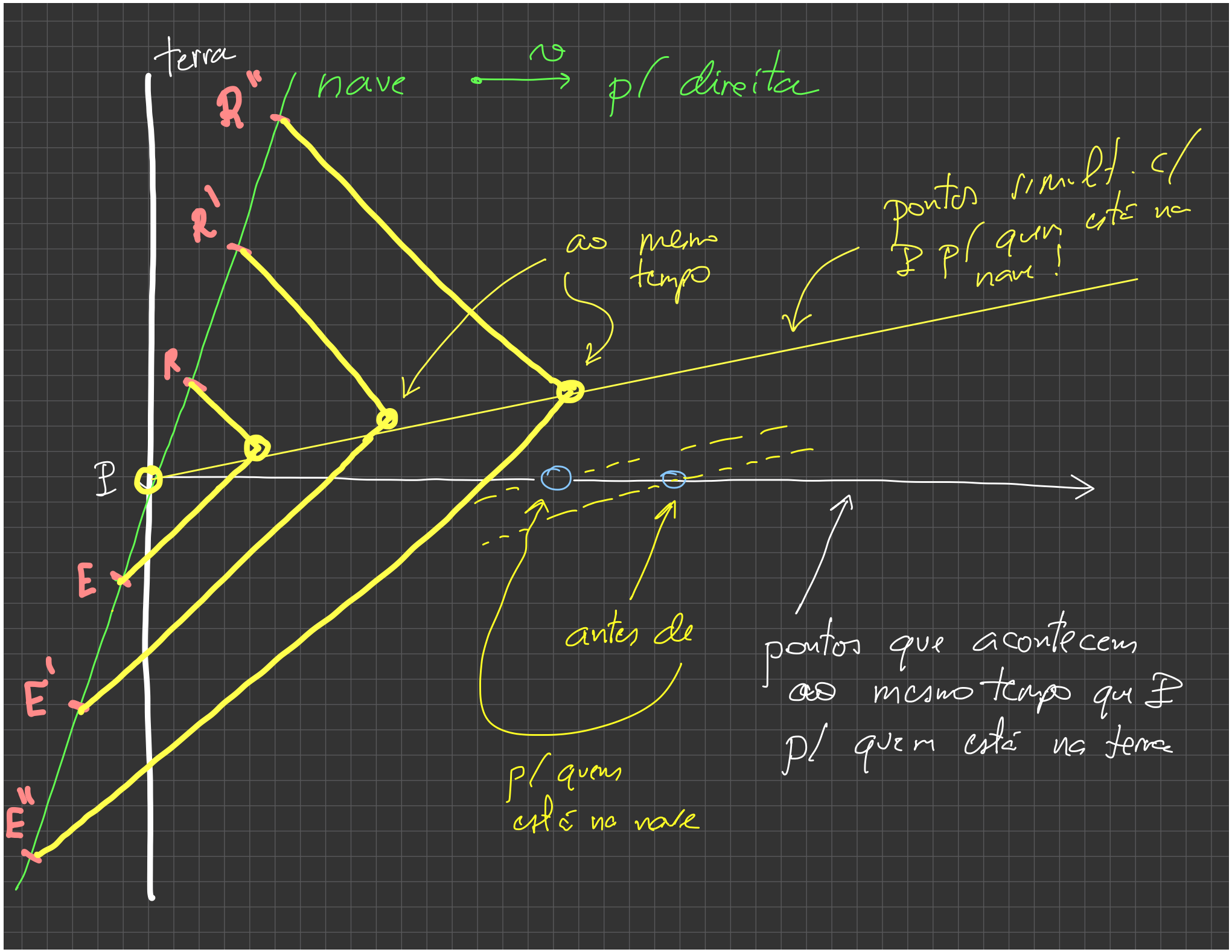
$$t = t_1 + \frac{t_2-t_1}{2} = \frac{t_1+t_2}{2}$$

na posição

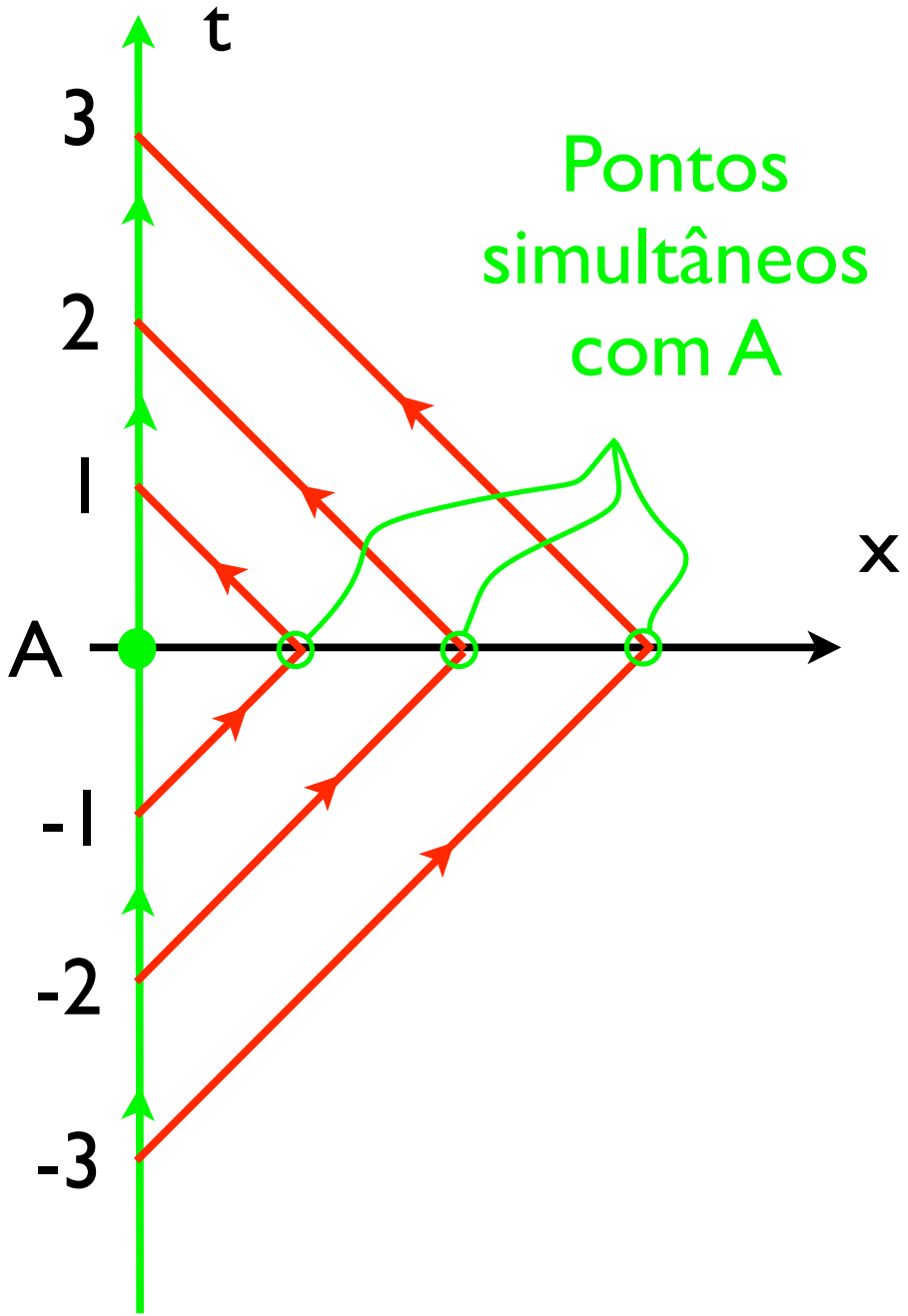
$$x = \frac{t_2-t_1}{2}\, c$$

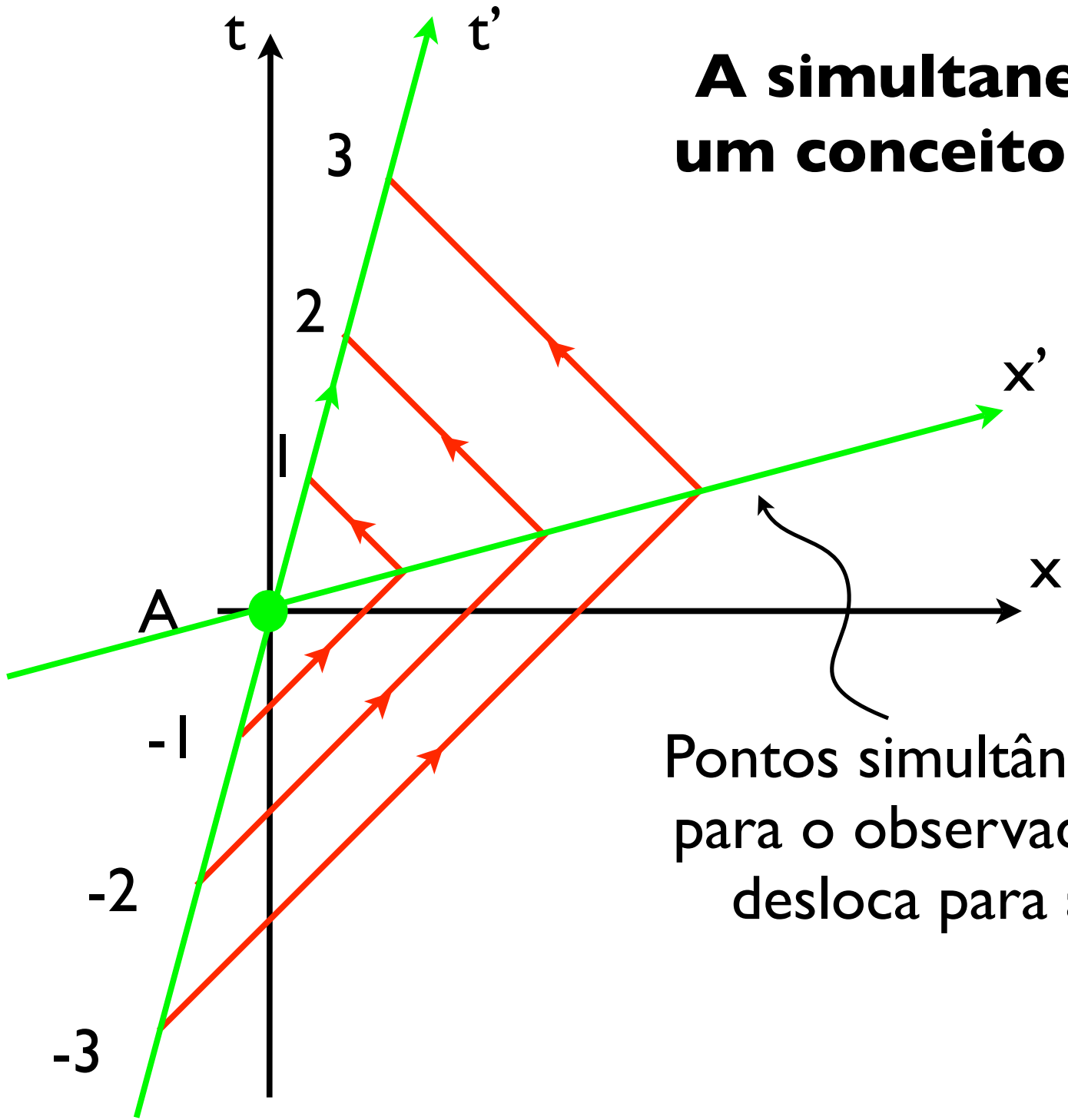
no tempo t e posição x


t
aconteceram ao mesmo tempo
≡ simultâneos
igual
a
alguém parado
x


terra
nave
v
p/ direita
R''
R'
R
P
E
E'
E''
ao mesmo tempo
pontos simult. c/ P p/ quem está na nave!
antes de
p/ quem está na nave
pontos que acontecem ao mesmo tempo que P p/ quem está na terra

na nave

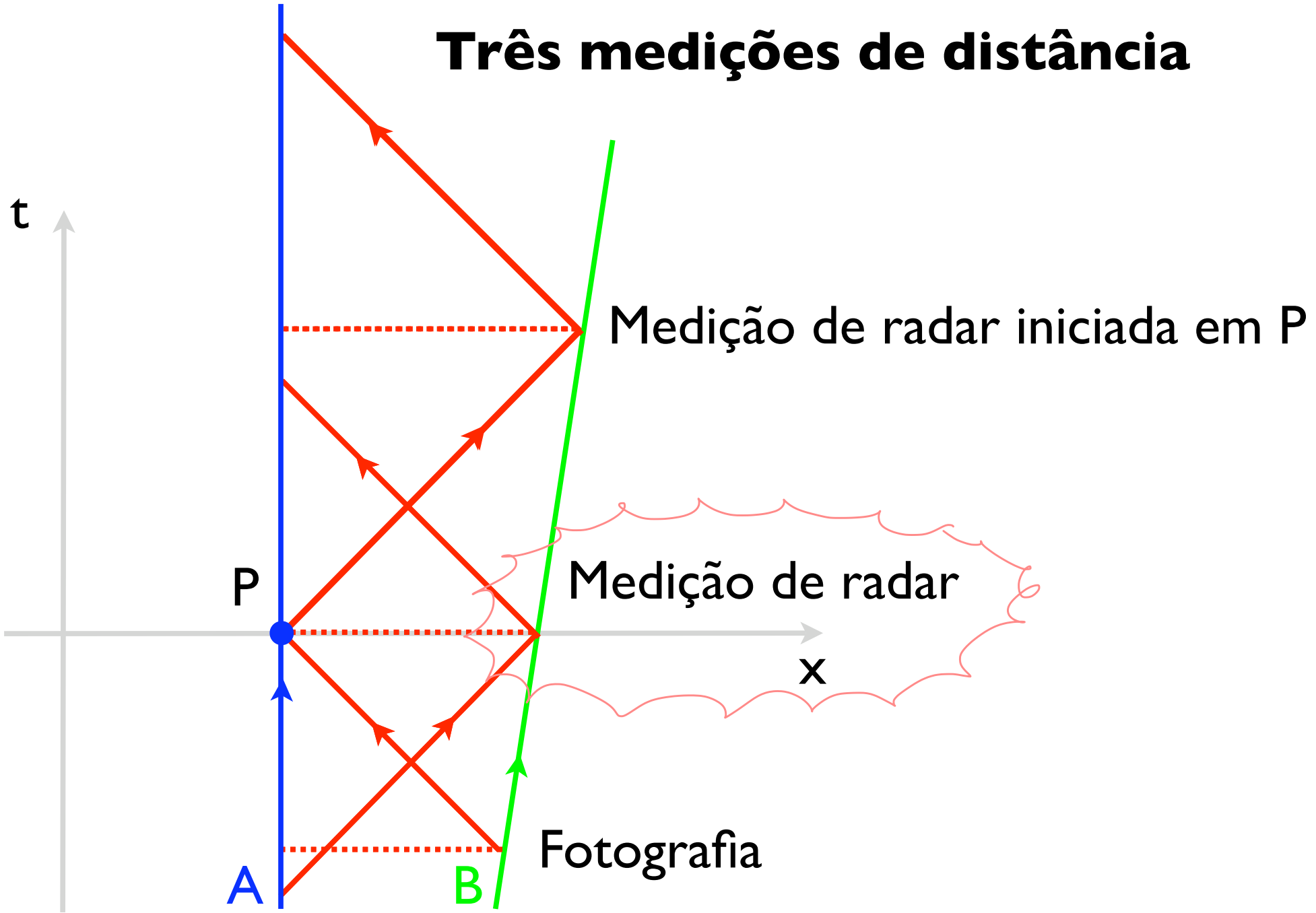




A simultaneidade é um conceito relativo

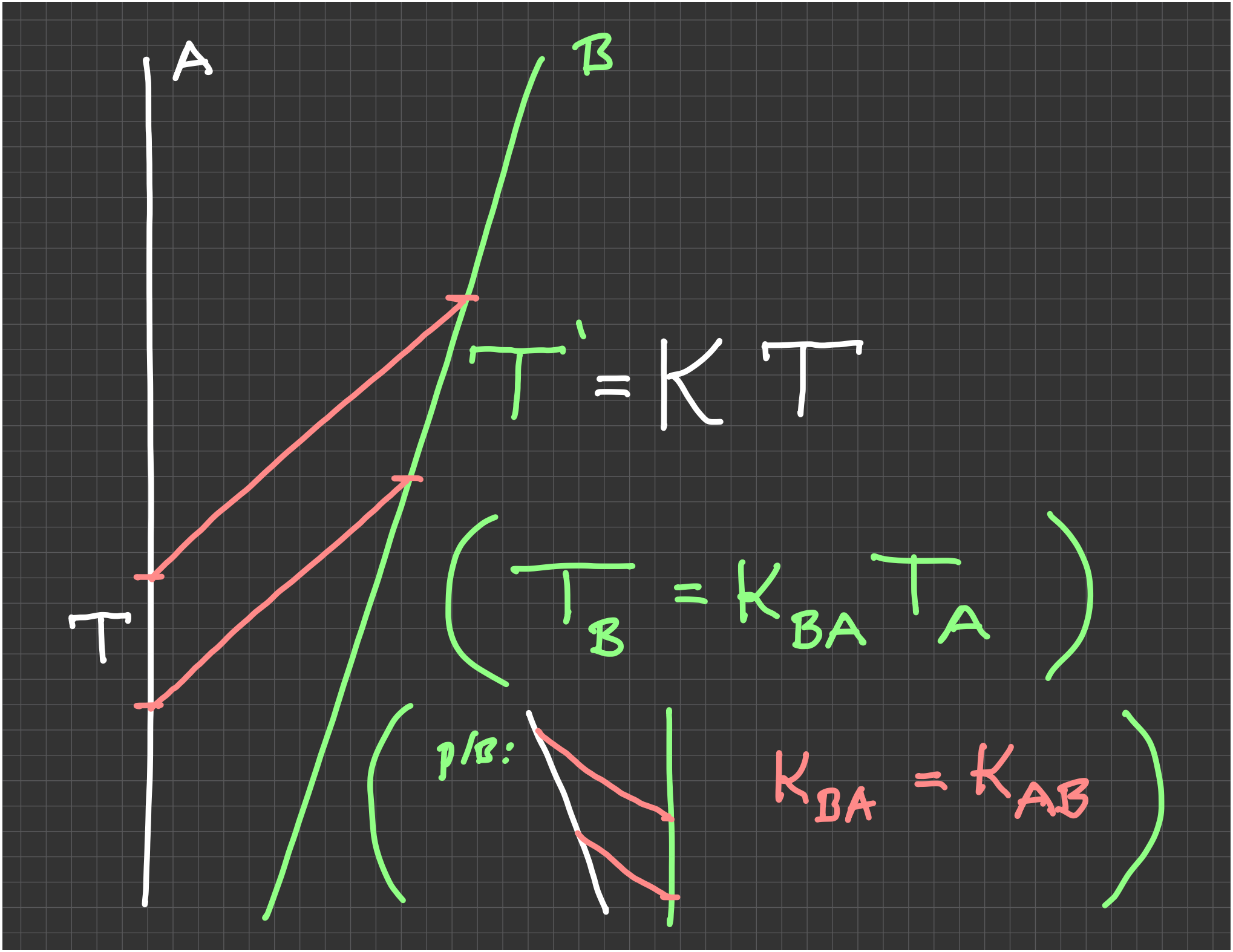
Pontos simultâneos com A para o observador que se desloca para a direita

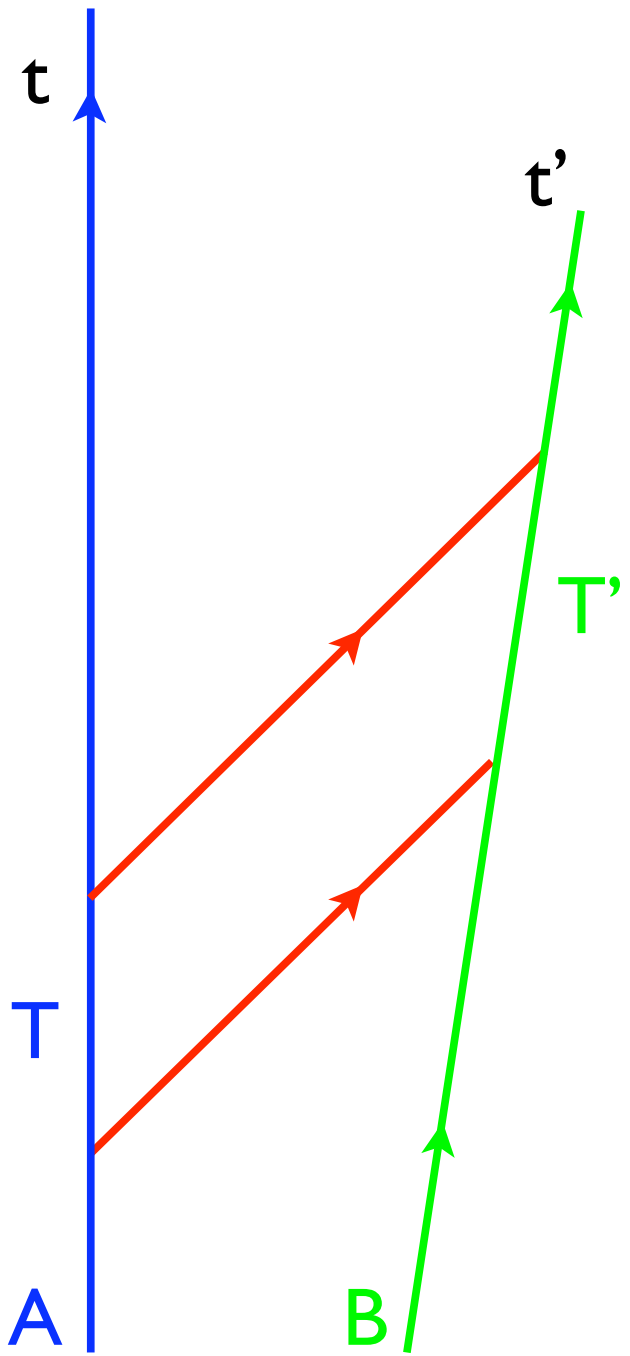
Três medições de distância



Cálculo K

nova demonstração da dil. do tempo.


A
B
T
T' = K T
(T_B = K_BA T_A)
p/B:
K_BA = K_AB



$K(v)$

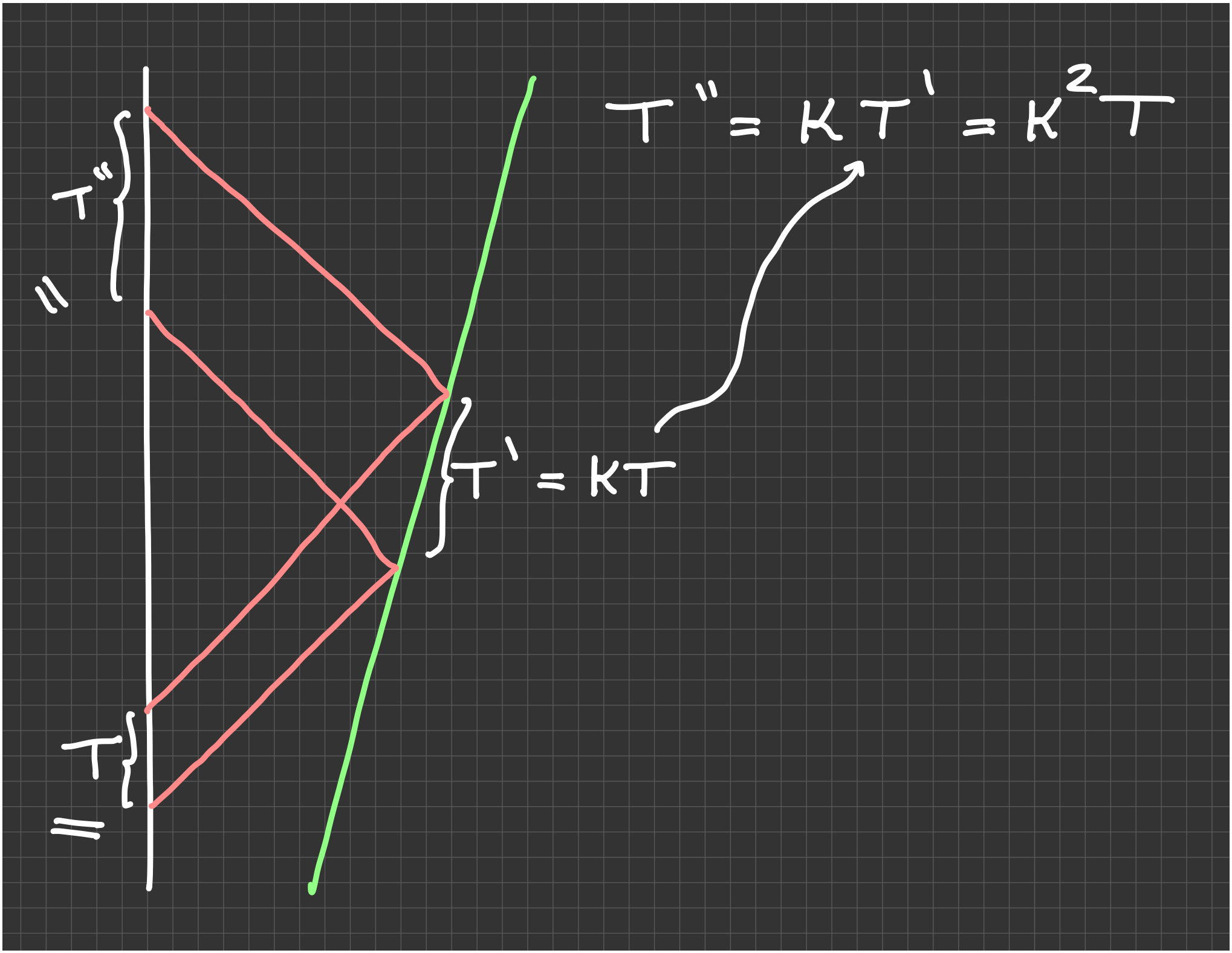
$$T' = K\,T$$

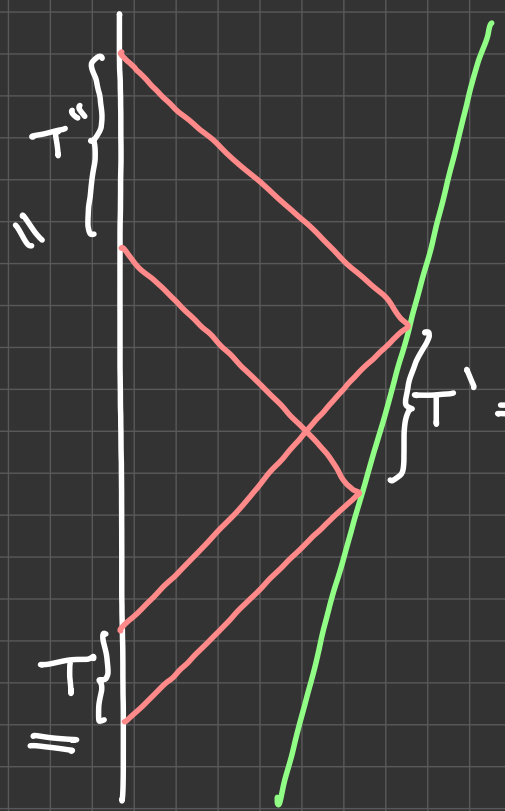
ou, melhor,

$$T_B = K_{BA}\,T_A$$

Princípio da Relatividade:

$$K_{AB} = K_{BA} = K$$

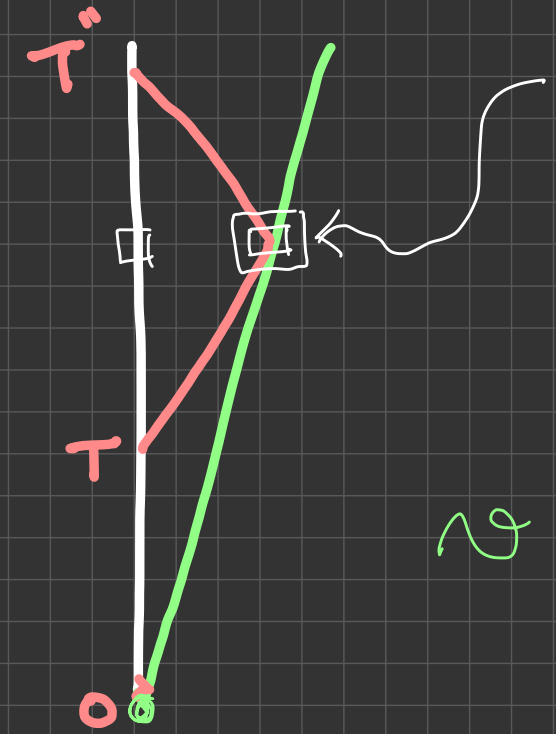

T''
T
T' = KT
T'' = KT' = K²T



$$T'' = KT' = K^2 T$$

$$\frac{K^2 - 1}{K^2 + 1} = \frac{v}{c}$$

$$K = \sqrt{\frac{1 + v/c}{1 - v/c}}$$



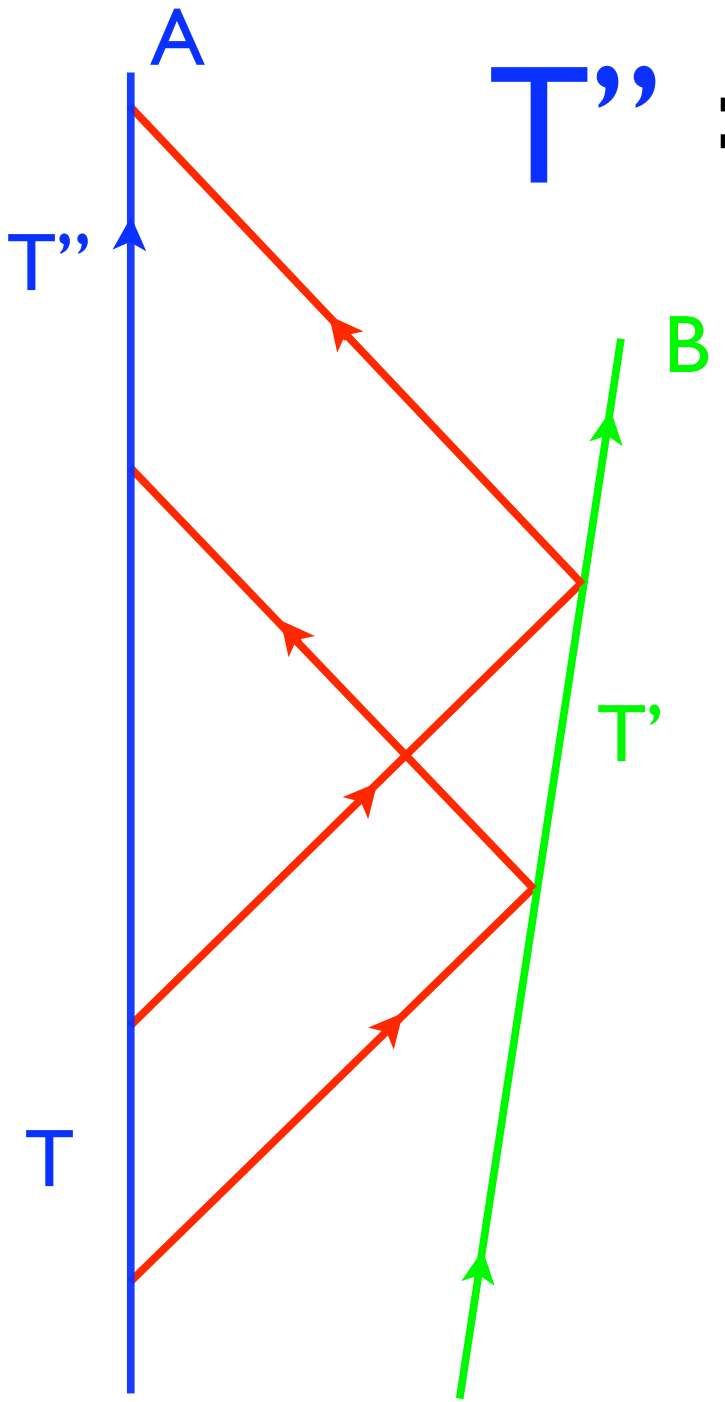
na terra: □ aconteceu no tempo $\frac{T'' + T}{2}$

a uma distância $\frac{T'' - T}{2} c$

tempo de ida

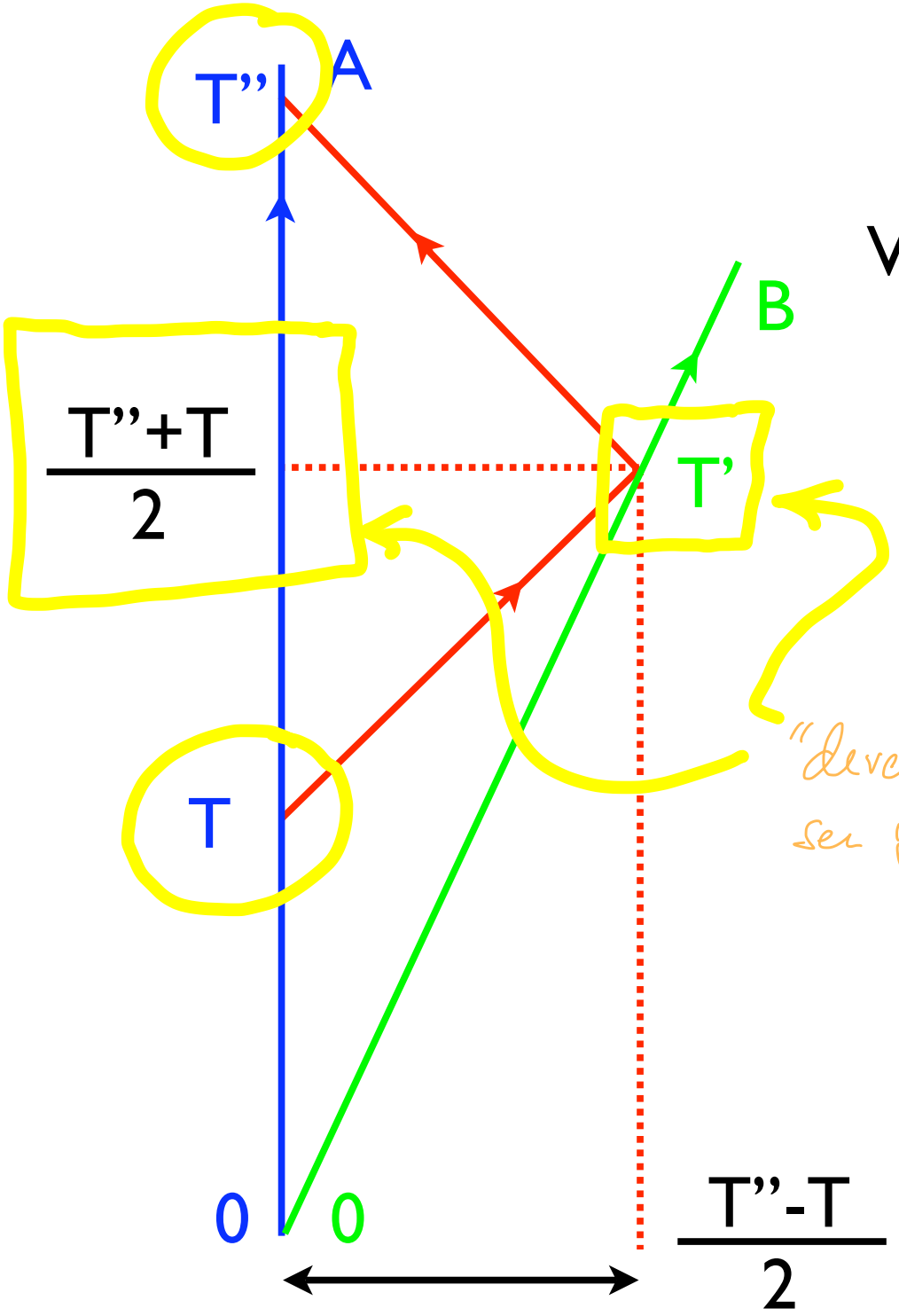
$$v = \frac{\frac{T'' - T}{2} c}{\frac{T'' + T}{2}}$$

$$v = \frac{K^2 - 1}{K^2 + 1} c$$



$$T'' \equiv KT' \equiv K(K\,T)$$

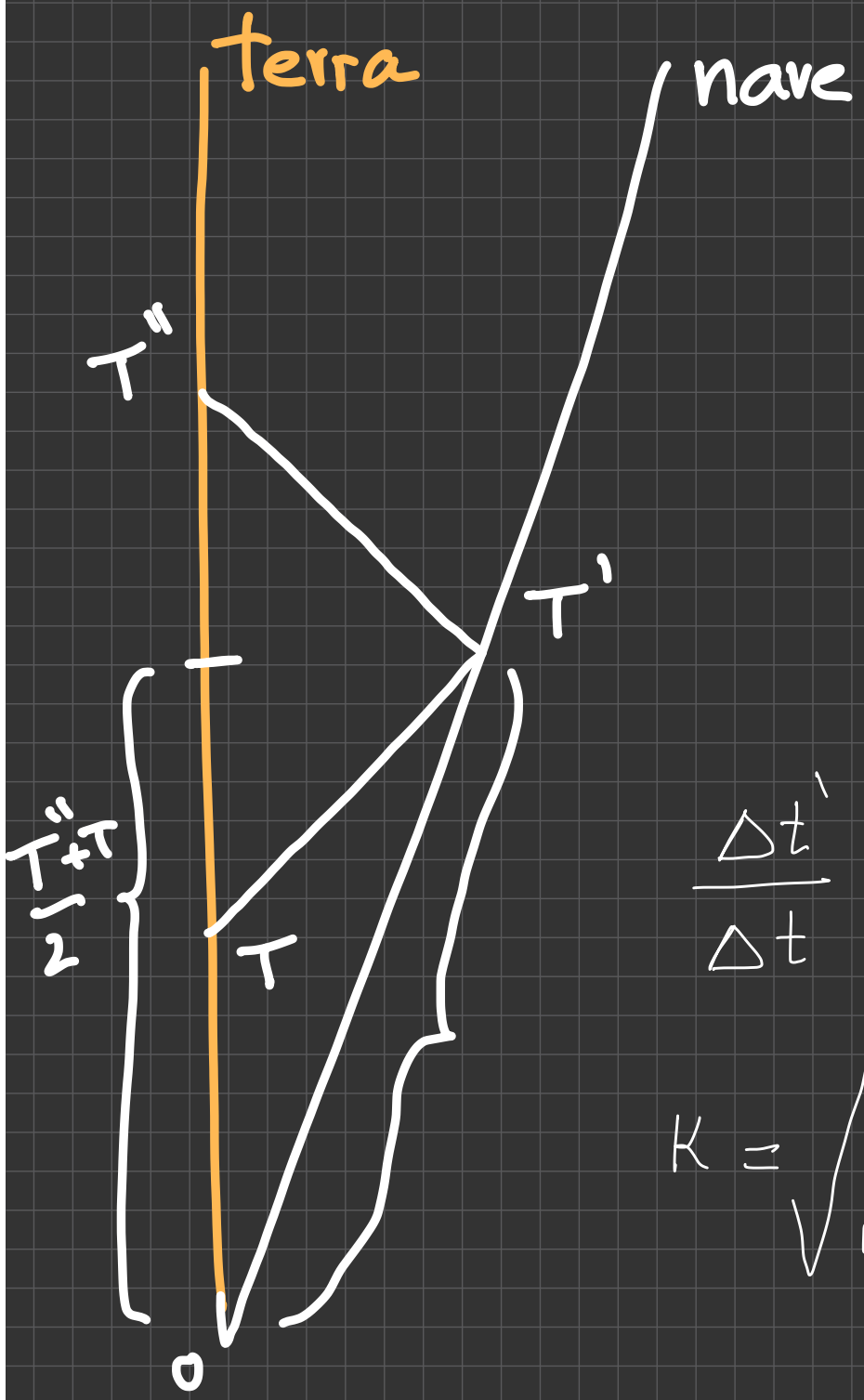
$$T'' = K^2\,T$$



$$V = \frac{\frac{T''-T}{2}\,c}{\frac{T''+T}{2}} = \frac{K^2 T - T}{K^2 T + T}\,c$$

$$\frac{V}{c} = \frac{K^2 - 1}{K^2 + 1}$$

$$K = \sqrt{\frac{1+V/c}{1-V/c}}$$



na nave

$$\Delta t = T'$$

$$\Delta t' = \frac{T''+T}{2}$$

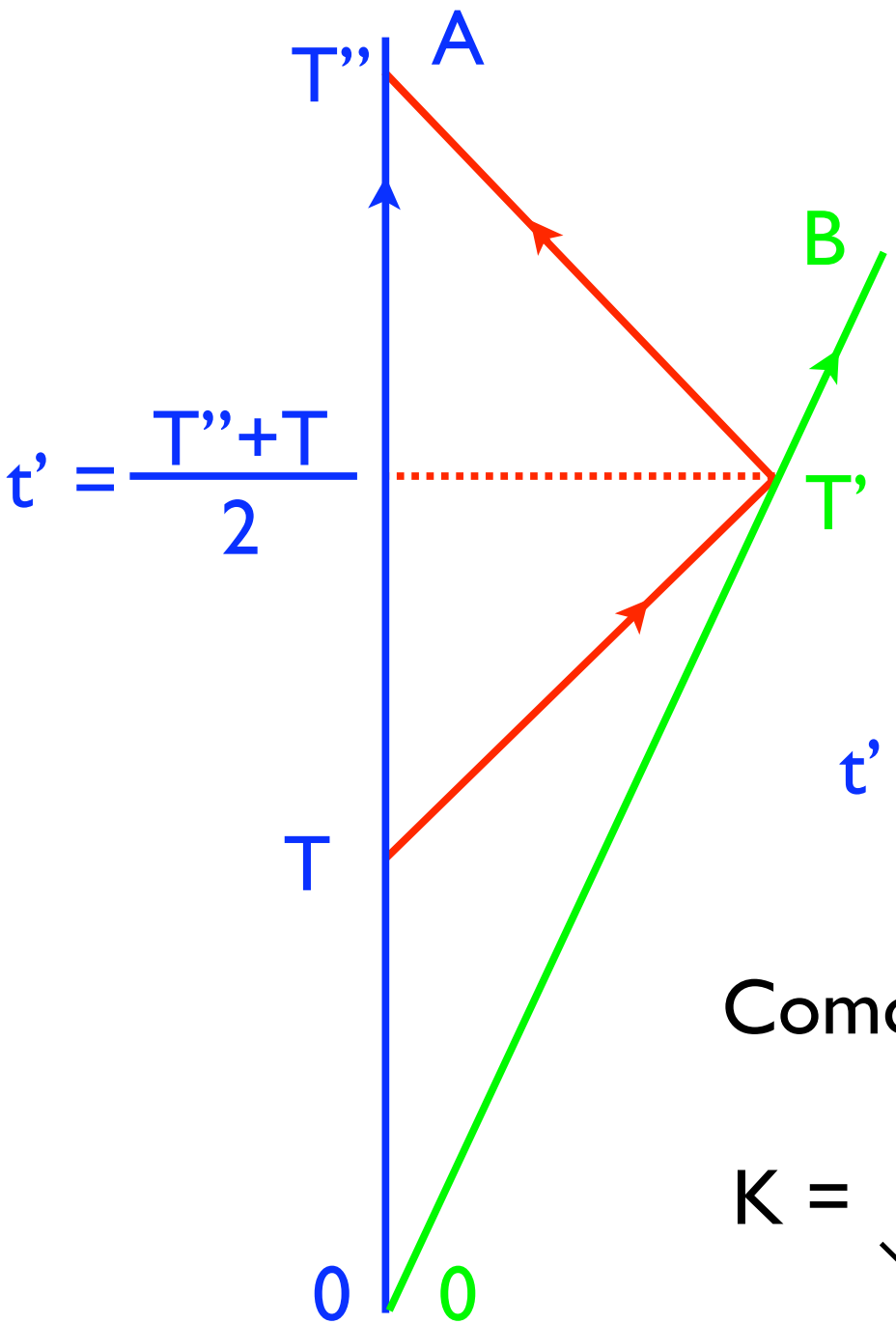
na terra

$$\frac{\Delta t'}{\Delta t} = \frac{T''+T}{2T'} = \frac{K^2T+T}{2KT} = \frac{K^2+1}{2K}$$

$$K = \sqrt{\frac{1+v/c}{1-v/c}}$$

$$\frac{\Delta t'}{\Delta t} = \frac{1}{\sqrt{1-v^2/c^2}}$$

Dilatação do Tempo



$T' = KT$ OK

$T'' = KT'$ OK

$t' = \frac{K^2 + 1}{2} T$ OK

$t' = \frac{K^2 + 1}{2K} t$ nada normal

Como

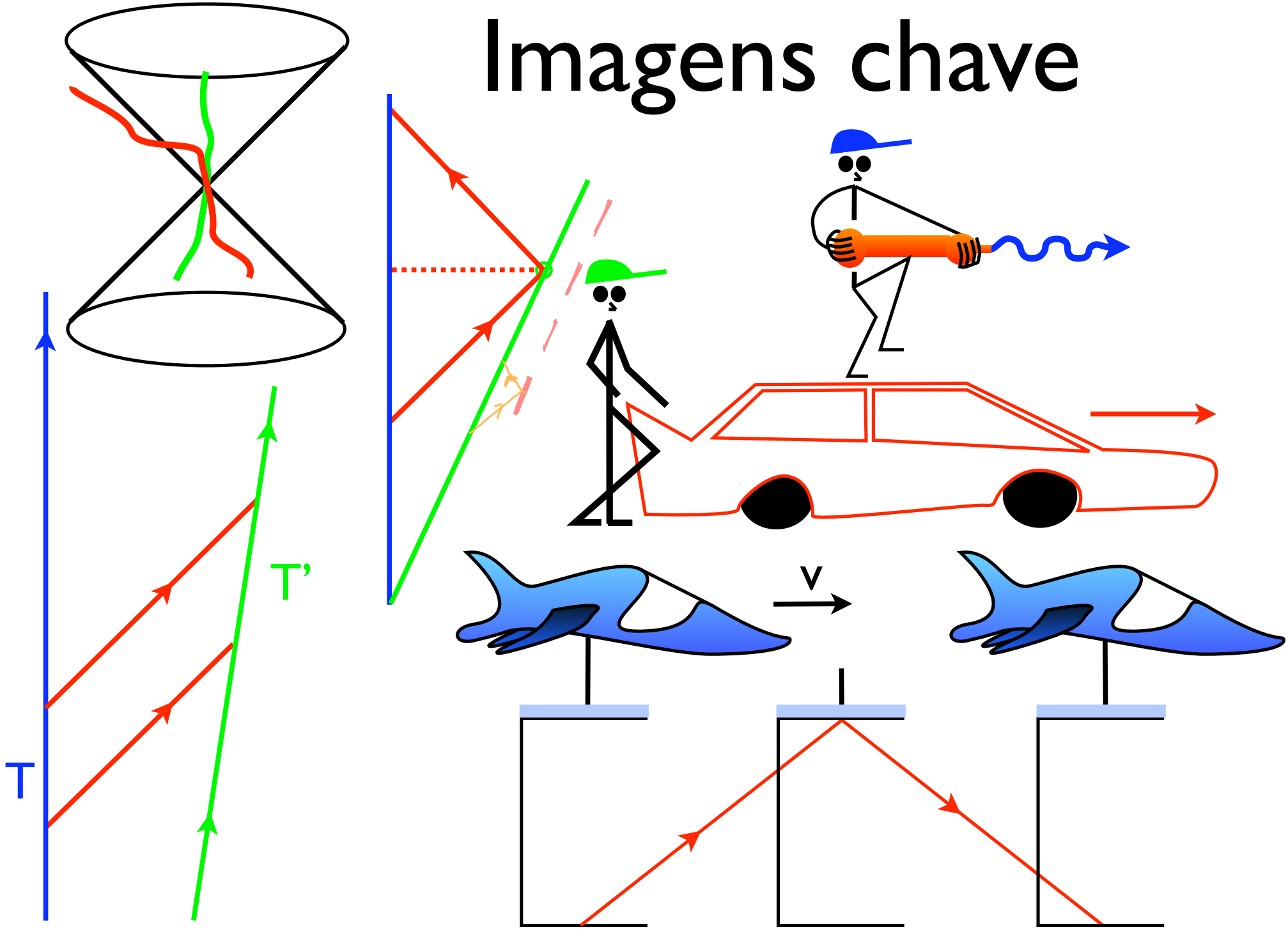
$K = \sqrt{\frac{1+V/c}{1-V/c}}$,

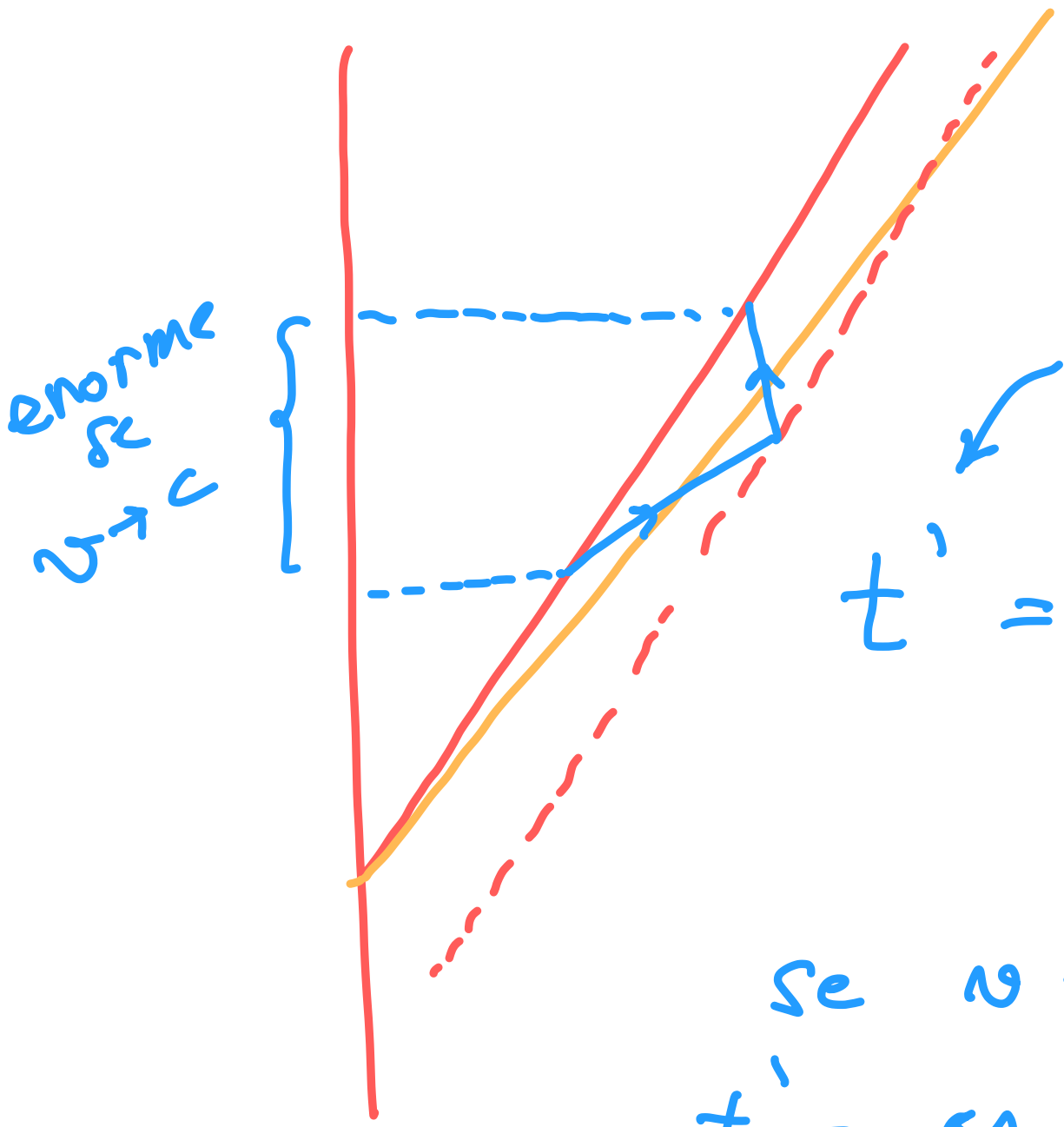
$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Palavras chave

- Diagramas de Espaço-Tempo
- Velocidade da Luz
- Cone de Luz
- Adição de velocidades
- Postulado
- Dilatação do Tempo
- Distâncias e Simultaneidade
- Cálculo K
- Dilatação do Tempo

Imagens chave





terra

nave

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

se $v \to c$, $\sqrt{\ } \to 0$

$$t' = \infty \times t$$